Rational Functions

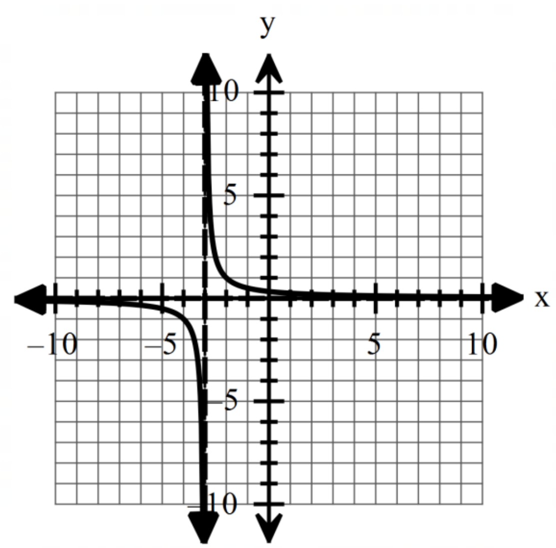
Date:

Objective:

Not only do rational graphs have end behavior, but they also have limits at the asymptotes.

Example: Evaluate the limit and end behavior based on the graph of $f(x)$ shown.

1.



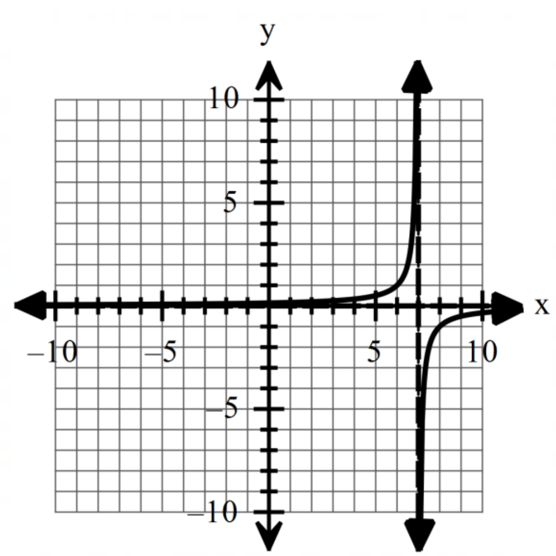
End Behavior:

$\lim_{x \to -\infty} f(x) =$ $\lim_{x \to \infty} f(x) =$

Limits at the asymptote:

$\lim_{x \to -3^-} f(x) =$ $\lim_{x \to -3^+} f(x) =$

2.



End Behavior:

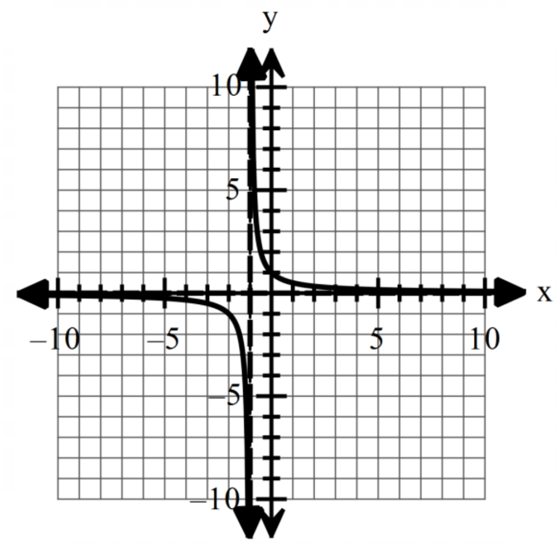
$\lim_{x \to -\infty} f(x) =$ $\lim_{x \to \infty} f(x) =$

Limits at the asymptote:

$\lim_{x \to 7^-} f(x) =$ $\lim_{x \to 7^+} f(x) =$

Example: Given the graphs of function below, determine the key features.

3.



Domain:

Range:

x-intercept(s):

y-intercept:

Increasing:

Decreasing:

Constant:

Vertical Asymptote(s):

Positive:

Negative:

Maximums / minimums:

Symmetry:

End Behavior/Limits:

$\lim_{x \to -\infty} f(x) =$ $\lim_{x \to \infty} f(x) =$

$\lim_{x \to -1^-} f(x) =$ $\lim_{x \to -1^+} f(x) =$

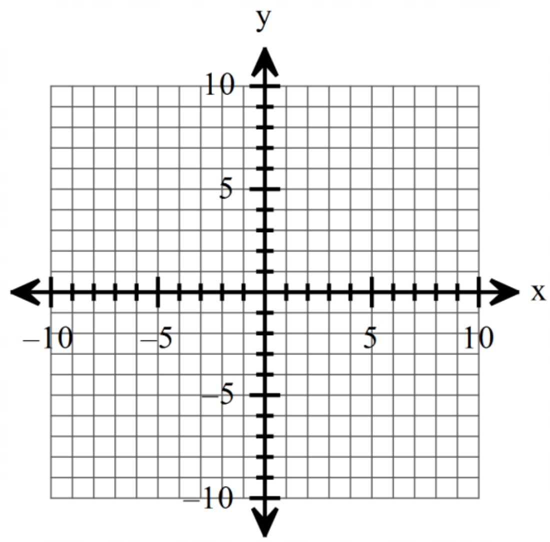
Horizontal Asymptote:

To graph a rational function, you need to find the horizontal and vertical asymptotes and the x- and y- intercepts. You will find the domain as well.

- To find the vertical asymptote, set the denominator equal to 0 and solve.
- To find the horizontal asymptote, we will always use $y = 0$. There are others but they are harder and we are doing the easy ones that will always be $y = 0$.
- To find the x-intercept set the equation equal to 0 and solve. Since we are always doing easy ones, there will never be an x-intercept.
- To find the y-itnercept, make all the x's 0 and evaluate. This will always be the constant of the numerator over the constant of the denominator.
- We have already found the domain in unit 4, but here is a review. Factor the denominator and set it $\neq 0$. Then solve. This will tell you what number you can't use in the domain. Then you can write the interval notation.

Example: Use the following information to graph the rational equations without technology and determine the domain.

4. $f(x) = \frac{1}{x+8}$



vertical asymptote: $\boldsymbol{x = -8}$

horizontal asymptote: $\boldsymbol{y = 0}$

x-intercept: **NONE**

y-intercept: $\boldsymbol{\left(0, \frac{1}{8}\right)}$

Domain:____________________________

Example: Find the vertical asymptotes (remember it is the same as the restrictions, set the denominator $=$ 0 and solve for x).

5. $f(x) = \frac{1}{x+5}$

Vertical Asymptote: _____________________

Example: Find the domain and write it in interval notation (remember it is the same as the vertical asymptote, set the denominator $\neq$ 0 and solve for x).

6. $f(x) = \frac{1}{x+5}$

Domain: _____________________

Example: Find the horizontal asymptotes and the x-intercept.

7. $f(x) = \frac{1}{x+5}$

Horizontal Asymptote: _____________________

x-intercept: _________________

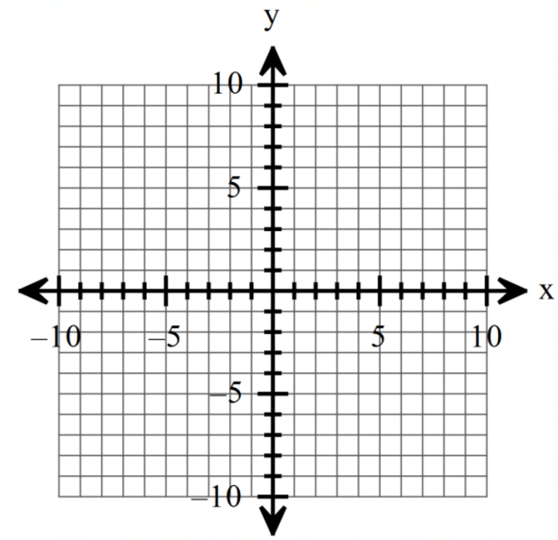
Example: Find the y-intercepts (make $x = 0$ and solve).

8. $f(x) = \frac{1}{x+5}$

y-intercept: _________________

Example: Graph each rational function without technology. You found all of the information in #5-8. Just copy it, don't find it again!

9. $f(x) = \frac{1}{x+5}$



Vertical Asymptote: ___________________

Domain: ________________________________

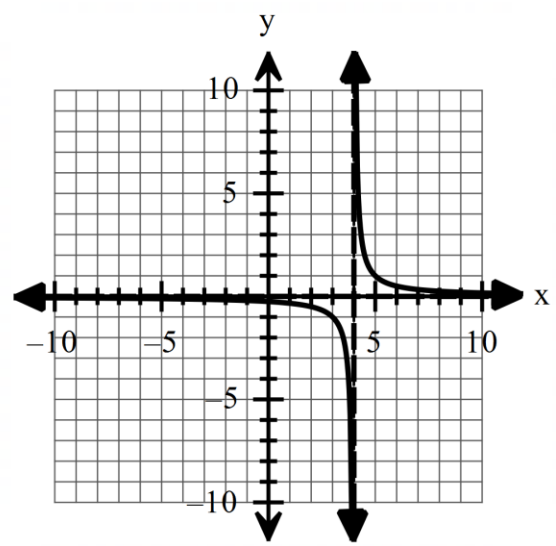
Horizontal Asymptote: ___________________

x-intercept: _____________________________

y-intercept: _____________________________

Example: Given the following graph, write an equation for the function.

10.



Equation: ____________________________