Rational Functions

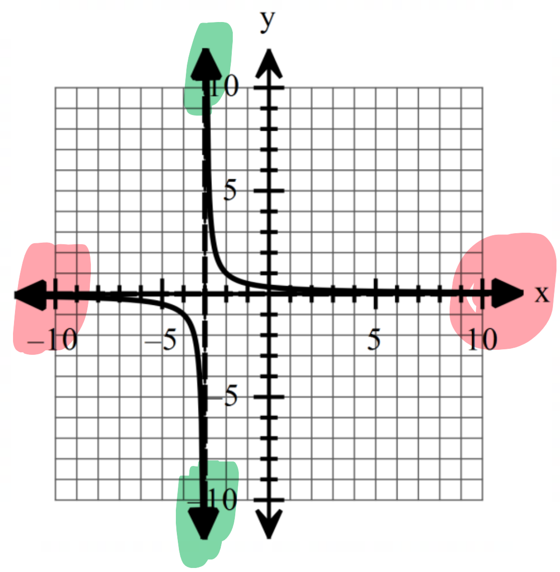
Date: 12/19/22

Objective: I can graph the reciprocal function which is a rational function.

Not only do rational graphs have end behavior, but they also have limits at the asymptotes.

Example: Evaluate the limit and end behavior based on the graph of $f(x)$ shown.

1.



End Behavior: = horizontal asym.

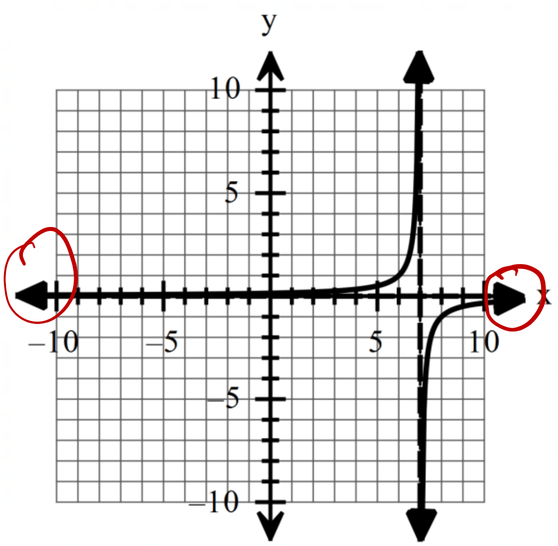
$\lim_{x\to-\infty} f(x) =$ 0 $\lim_{x\to\infty} f(x) =$ 0

Limits at the asymptote:

$\lim_{x\to-3^-} f(x) =$ $-\infty$ $\lim_{x\to-3^+} f(x) =$ ∞

left side right side

2.



End Behavior:

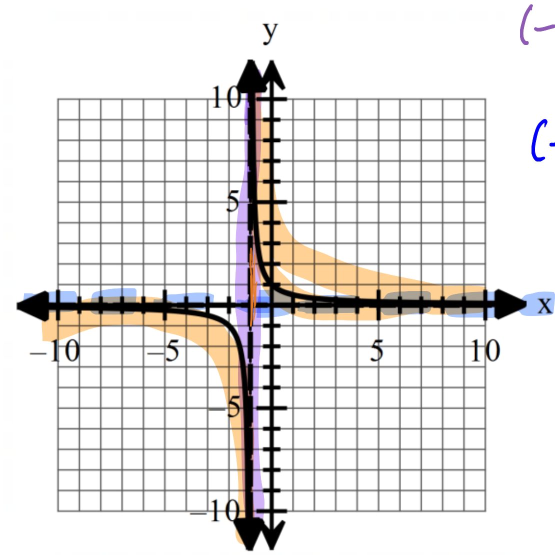
$\lim_{x\to-\infty} f(x) =$ 0 $\lim_{x\to\infty} f(x) =$ 0

Limits at the asymptote:

$\lim_{x\to\cancel{-}7^-} f(x) =$ ∞ $\lim_{x\to\cancel{-}7^+} f(x) =$ $-\infty$

Example: Given the graphs of function below, determine the key features.

3.



Domain: $(-\infty,-1)\cup(-1,\infty)$

Range: $(-\infty,0)\cup(0,\infty)$

x-intercept(s): none

y-intercept: $(0,1)$

Increasing: N/A

Decreasing: $(-\infty,-1)\cup(-1,\infty)$

Constant: N/A

Positive: $(-1,\infty)$

Negative: $(-\infty,-1)$

Maximums / minimums: N/A

Symmetry: N/A

End Behavior/Limits:

$\lim_{x\to-\infty} f(x) = 0$ $\quad \lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-1^-} f(x) = -\infty$ $\quad \lim_{x\to-1^+} f(x) = \infty$

Vertical Asymptote(s): $x=-1$

Horizontal Asymptote: $y=0$

To graph a rational function, you need to find the horizontal and vertical asymptotes and the x- and y- intercepts. You will find the domain as well.

- To find the vertical asymptote, set the denominator equal to 0 and solve. restrictions
- To find the horizontal asymptote, we will always use $y = 0$. There are others but they are harder and we are doing the easy ones that will always be $y = 0$.
- To find the x-intercept set the equation equal to 0 and solve. Since we are always doing easy ones, there will never be an x-intercept.
- To find the y-itnercept, make all the x's 0 and evaluate. This will always be the constant of the numerator over the constant of the denominator.
- We have already found the domain in unit 4, but here is a review. Factor the denominator and set it $\neq 0$. Then solve. This will tell you what number you can't use in the domain. Then you can write the interval notation.

Example: Use the following information to graph the rational equations without technology and determine the domain.

4. $f(x) = \frac{1}{x+8}$

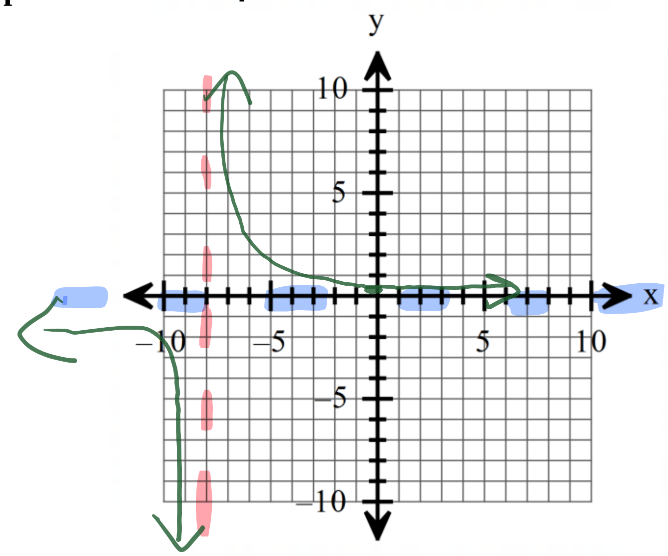
vertical asymptote: $\boldsymbol{x = 8}$

horizontal asymptote: $\boldsymbol{y = 0}$

x-intercept: **NONE**

y-intercept: $\left(0, +\frac{1}{8}\right)$ $\frac{1}{0+8}$

Domain: $(-\infty, -8) \cup (-8, \infty)$



Example: Find the vertical asymptotes (remember it is the same as the restrictions, set the denominator = 0 and solve for x).

5. $f(x) = \frac{1}{x+5}$ Vertical Asymptote: $x = -5$

Example: Find the domain and write it in interval notation (remember it is the same as the vertical asymptote, set the denominator ≠ 0 and solve for x).

6. $f(x) = \frac{1}{x+5}$ Domain: $(-\infty, -5) \cup (-5, \infty)$

Example: Find the horizontal asymptotes and the x-intercept.

7. $f(x) = \frac{1}{x+5}$ Horizontal Asymptote: $y = 0$

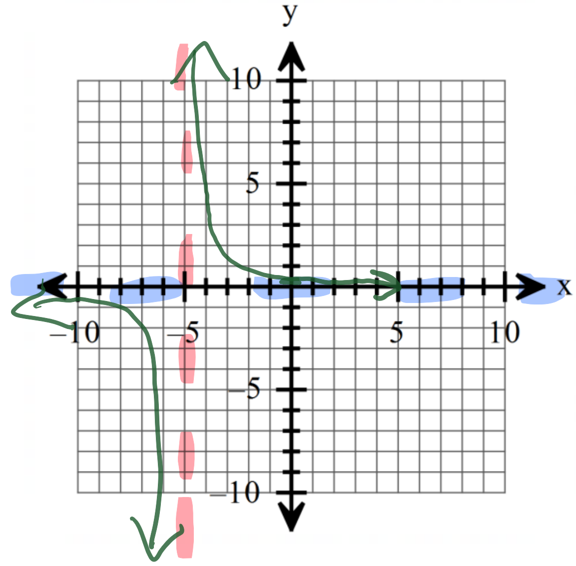
x-intercept: none

Example: Find the y-intercepts (make $x = 0$ and solve).

8. $f(x) = \frac{1}{x+5}$ y-intercept: $\left(0, \frac{1}{5}\right)$

Example: Graph each rational function without technology. You found all of the information in #5-8. Just copy it, don't find it again!

9. $f(x) = \frac{1}{x+5}$



Vertical Asymptote: $x=-5$

Domain: $(-\infty,-5)\cup(-5,\infty)$

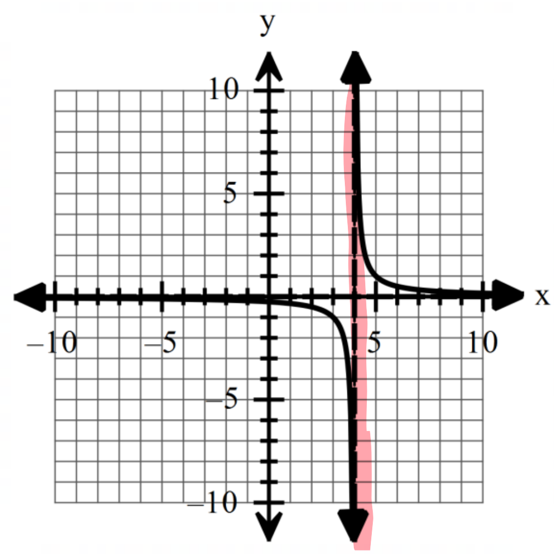
Horizontal Asymptote: $y=0$

x-intercept: None

y-intercept: $(0,\frac{1}{5})$

Example: Given the following graph, write an equation for the function.

10.



Equation: $f(x)=\frac{1}{x-4}$

$x=4$