11.3 Date: 4/30/24

Objective: I can graph exponential functions.

A. Warm-up: Practice ***Laws of Exponents***.

$a^s \cdot a^t =$ $\qquad$ $(a^s)^t =$ $\qquad$ $(ab)^s =$ $\qquad$ $1^s =$ $\qquad$ $a^{-s} =$ $\qquad$ $a^0 =$

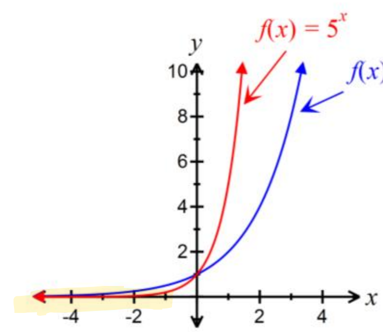
B. Properties of Exponential Functions

An ***exponential function*** is a function of the form $y = a^x$ where *a* is a positive real number $(a > 0)$ and $a \neq 1$. The domain of f is the set of all real numbers.

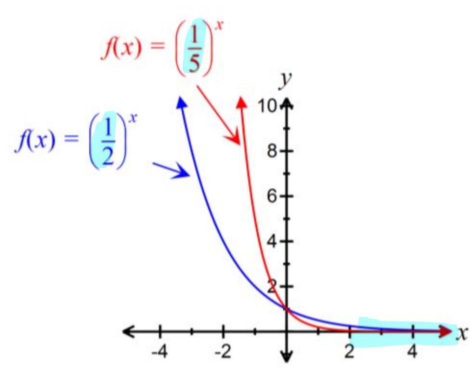
Properties of the Exponential Function $f(x) = a^x,\ a > 0,\ a \neq 1$

- Domain: $(-\infty, \infty)$ Range: $(0, \infty)$
- There are no x-int; the *y*-intercept is (0, 1).
- The *x*-axis $(y = 0)$ is a asymptote.
    - For $a > 1$, the graph approaches the *x*-axis as x is smaller
    - For $0 < a < 1$, the graph approaches the *x*-axis as x is bigger
- $f(x) = a^x$ is one-to-one.
    - For $a > 1$, $f(x) = a^x$ is an increasing function.
    - For $0 < a < 1$, $f(x) = a^x$ is a decreasing function.
- The graph of f contains the points $(-1, \frac{1}{a})$, $(0, 1)$, and $(1, a)$.

$a > 1$



$0 < a < 1$



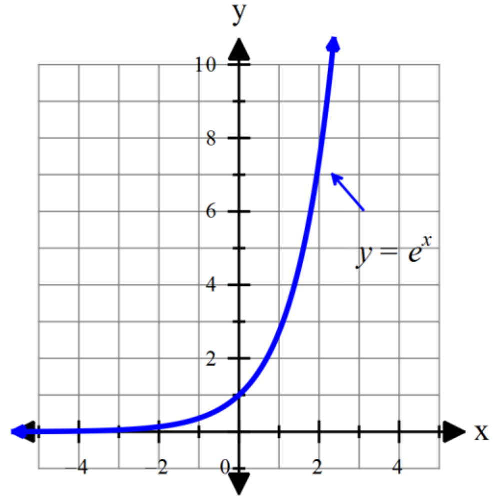
Asymptote:

Asymptote Equation: $y = k$

C. The number e

- The **number *e*** (approximately 2.71828…) is defined as the number that the expression $\left(1+\frac{1}{n}\right)^n$ approaches as $n \to \infty$. In calculus, this is expressed using limit notation as $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.

- Find e^2



D. Review Transformations. (No Calculators!)

- The general equation for an exponential function is: $y = b \cdot a^{c(x-h)} + k$
  List the transformation that corresponds with each variable.

  b =________________ c = ______________ h = _______________ k = _______________

  Negative function ____________________ Negative exponent ____________________

- Without a Calculator, match each equation to the appropriate graph.

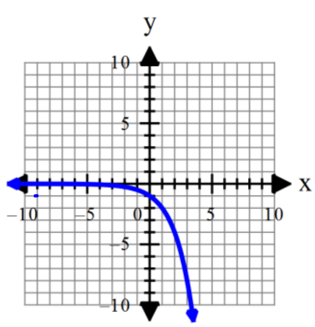
$a)\ y = 2^x$ 6 $b)\ y = -2^x$ 1 $c)\ y = 2^{-x}$ 5 $d)\ y = 2^x - 1$ 3

$(c(x-h))$

$e)\ y = -2^{-x}$ 2 $f)\ y = 2^{x-1}$ 4 $g)\ y = 1 - 2^x$ 7 $h)\ y = 2^{1-x}$ 8 $y = 2^{-x+1} = 2^{-(x-1)}$

$y = -2^x + 1$

1.



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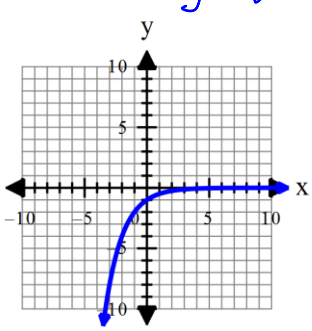

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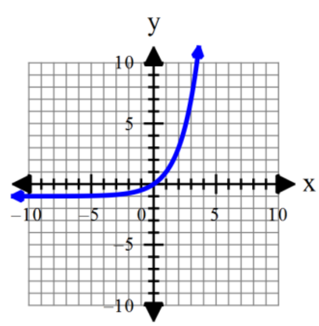

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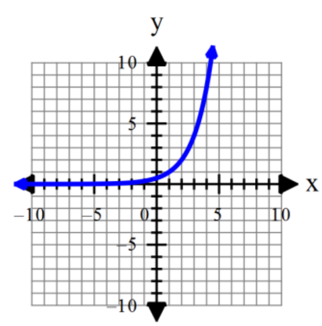

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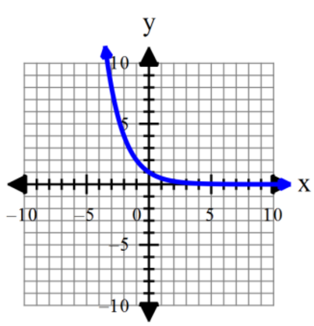

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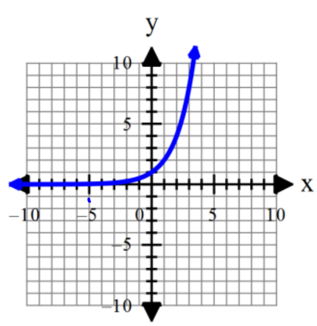

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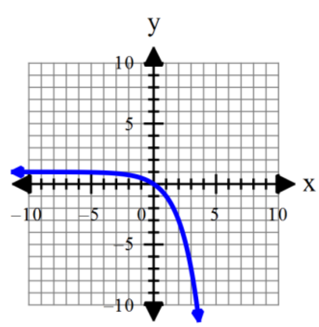

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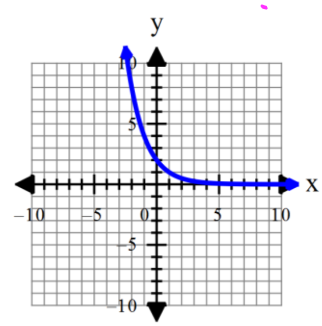
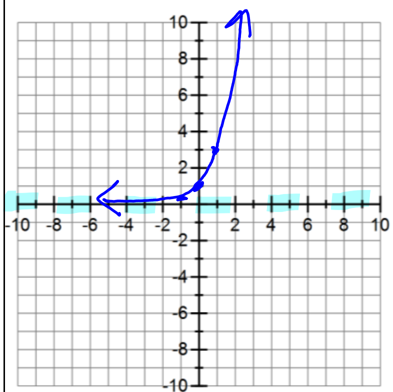


E. Graphing using transformations and 3 key points.

Examples: Use 3 key points and transformations to graph. (No Calculators!) Find domain, range, and horizontal asymptote.

a) Graph $f(x) = 3^x. + 0$

a = 3



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal asymptote: $y = 0$

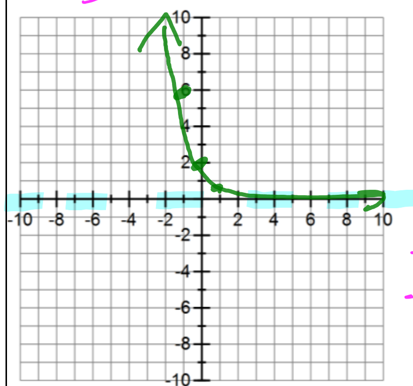
Key points and transformations:

x	y
-1	$\frac{1}{3}$
0	1
1	3

b) Graph $f(x) = 2\left(\frac{1}{3}\right)^x. + 0$

a = $\frac{1}{3}$

b=vert strtch



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal asymptote: $y = 0$

Key points and transformations:

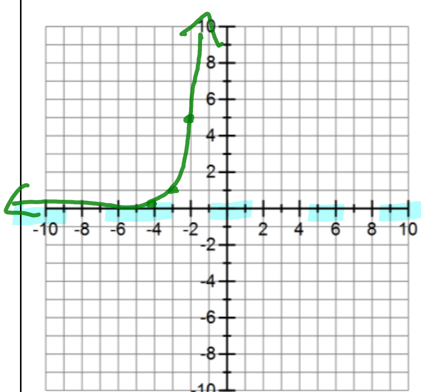
x	y
-1	3
0	1
1	$\frac{1}{3}$

$\rightarrow$

x	$2y$
-1	6
0	2
1	$\frac{2}{3}$

c) Graph $f(x) = 5^{x+3}. + 0$

a = 5



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal asymptote: $y = 0$

Key points and transformations:

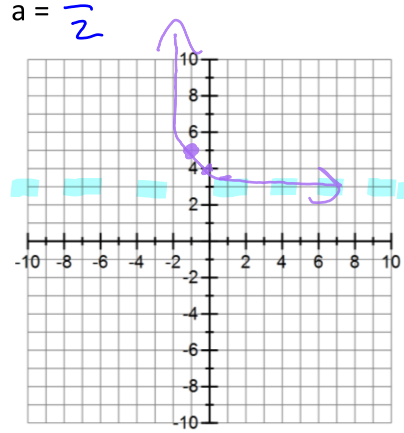
x	y
-1	$\frac{1}{5}$
0	1
1	5

$\rightarrow$

$x-3$	y
-4	$\frac{1}{5}$
-3	1
-2	5

d) Graph $f(x) = \left(\frac{1}{2}\right)^x + 3.$

a = $\frac{1}{2}$



Domain:

Range: $(3, \infty)$

Horizontal asymptote: $y = 3$

Key points and transformations:

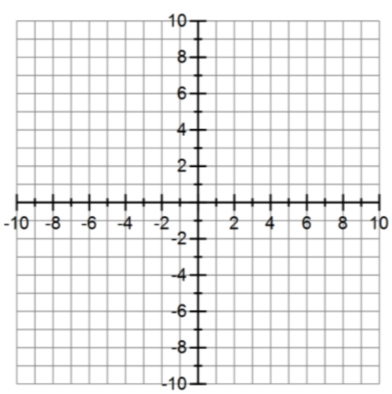
x	y
-1	2
0	1
1	$\frac{1}{2}$

$\rightarrow$

x	$y+3$
-1	5
0	4
1	$3\frac{1}{2}$

e) Graph $f(x) = 2^{-x}.$

a =



Domain:

Range:

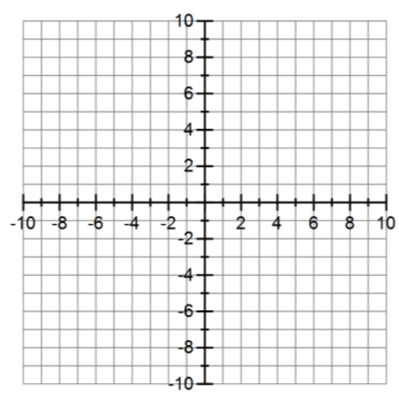
Horizontal asymptote:

Key points and transformations:

$-x$	y

f) Graph $f(x) = -3^x$

a =



Domain:

Range:

Horizontal asymptote:

Key points and transformations:

x	$-y$

F. Review finding domain algebraically

Domain is always $(-\infty,\infty)$ unless there is a restriction.

The 3 types of restrictions are:

1. even roots ($\sqrt{\ }$) $\sqrt{\ } \geq 0$
2. rational $\frac{1}{x}$ $x \neq 0$ denom $\neq 0$
3.

EXAMPLES:

a. $f(x)=3x+4$

$(-\infty,\infty)$

b. $f(x)=5x^2-13x-6$

$(-\infty,\infty)$

c. $f(x)=5\sqrt{x+4}$

$x+4\geq 0$
$x\geq -4$
$[-4,\infty)$

d. $f(x)=\dfrac{5}{x+2}$ $\neq 0$

$x\neq -2$

$(-\infty,-2)\cup(-2,\infty)$

e. $f(x)=\dfrac{2x-3}{4x-1}$ $\neq 0$

$4x\neq 1$

$x\neq \frac{1}{4}$

$(-\infty,\frac{1}{4})\cup(\frac{1}{4},\infty)$

f. $f(x)=(x+4)^3-8$

$(-\infty,\infty)$

G. Stories

When the interest due at the end of a payment period is added to the principal so that the interest computed at the end of the next payment period is based on the new principal amount (old principal plus interest), the interest is said to have been ***compounded***. ***Compound interest*** is interest paid on principal and previously earned interest.

Compound Interest Formula

The amount A after t years due to a principal P invested at an annual interest rate t compounded n times per year is $\boldsymbol{A=P\cdot\left(1+\frac{r}{n}\right)^{nt}}$.

April 1 – April 30   May 1 20%–25%
$1000   interest $250 $1250

Continuous Compounding

The amount A after t years due to a principal P invested at an annual interest rate r compounded continuously is $\boldsymbol{A=Pe^{rt}}$.

Example: Investing $1000 at an annual rate of 9% (.09) compounded annually, semiannually, quarterly, monthly, and daily will yield the following amounts after 1 year:

Annual Compounding (n = 1):

Semiannual Compounding (n = 2):

Quarterly Compounding (n = 4):

$1000\left(1+\frac{.09}{4}\right)^{4\cdot 1}\approx \$1093.08$

Monthly Compounding (n = 12):

$1000\left(1+\frac{.09}{12}\right)^{12\cdot 1}\approx \$1093.81$

Daily Compounding (n = 365):

Continuously Compounding (use e):

$1000e^{.09(1)}\approx \$1094.17$