11.4

Date: **Section:**

Objective:

A. Inverses of exponential functions.

- The logarithmic function $y = \log_a x$ is the inverse of the exponential function ________.
- Domain $y = a^x$: ________. Range $y = a^x$:________.
- Domain $y = \log_a x$: __________. Range $y = \log_a x$:__________.

Domain of the logarithmic function =

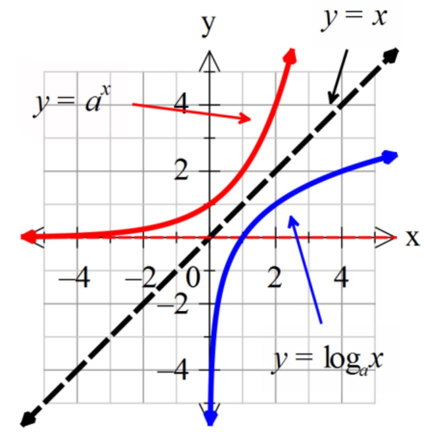
________ of the exponential function $= (0, \infty)$

Range of the logarithmic function =

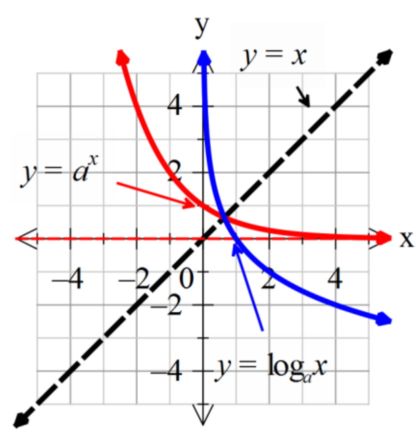
__________ of the exponential function $= (-\infty, \infty)$

★ **Caution!** You can't take the log of zero or of a negative because it is impossible to get zero or a negative by raising a positive base to an exponent. **The argument of a logarithmic function must be greater than zero.**

$a > 1$

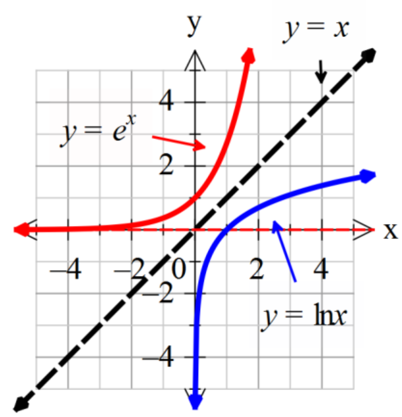


$0 < a < 1$



Properties of the Logarithmic Function $f(x) = \log_a x$

- The x-intercept is ______. There is ______ y-intercept.
- The vertical asymptote of the graph is __________________.
- The logarithmic function is _____________ if $0 < a < 1$ and _______________ if $a > 1$. The function is one-to-one.
- Since $y = \log_a x$ is the inverse of $x = a^y$ and the graph $x = a^y$ contains the points $\left(-1, \frac{1}{a}\right)$, $(0,1)$, and $(1, a)$ then the graph of $y = \log_a x$ contains the points (____,____),(____,____), and (____,____).



Common Logarithmic Function: If the base of a logarithmic function is the number 10, then we have the common logarithmic function. If the base a of the logarithmic function is not indicated, it is understood to be 10. That is, $y = \log x$ if and only if $x = 10^y$.

Natural Logarithms: If the base of a logarithmic function is the number e, then we have the natural logarithm function (abbreviated ln). That is, $y = \ln x$ if and only if $x = e^y$.

$y = \ln x$ is the ____________of $y = e^x$

Asymptote Equation:

B. Finding the domain of logarithmic functions.

1. $f(x) = \log_2(x+3)$
2. $h(x) = -\log_{\frac{1}{2}} x$
3. $g(x) = \ln(-x-5)+3$

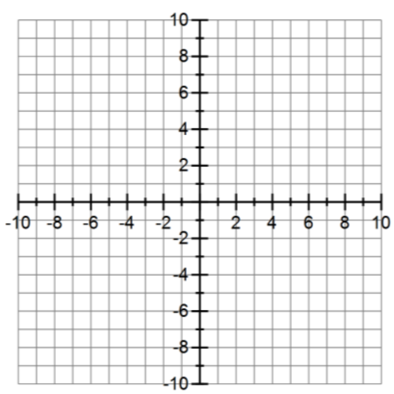
C. Graphing logarithmic functions.

Steps for Graphing Logarithmic Functions:

1. Find the domain
2. Find the asymptotes
3. Graph the asymptotes
4. Find the 3 key points $(1,0)$, $(a,1)$, **and** $\left(\frac{1}{a},-1\right)$ and apply the appropriate transformations.
5. Plot your points and connect them to form a smooth curve.
6. Find the range

Examples: Graph the following functions.

a) $y = \log_2 x$



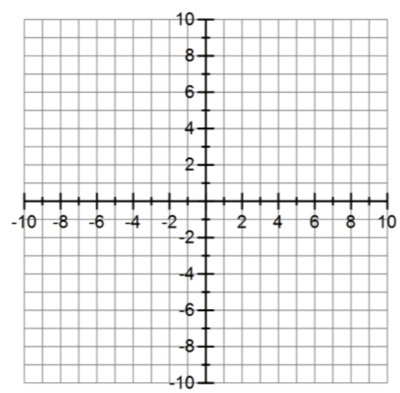
Domain:

Asymptotes:

Key points and transformations:

Range:

b) $y = \log(-x) - 2$.



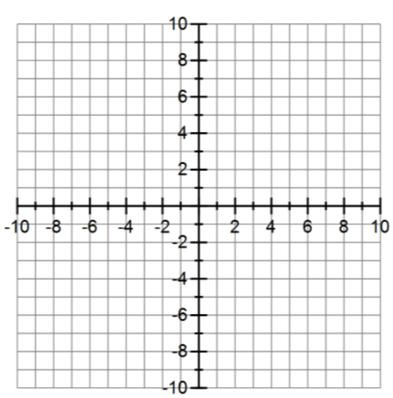
Domain:

Asymptotes:

Key points and transformations:

Range:

c) $f(x) = -\ln(x+3)$



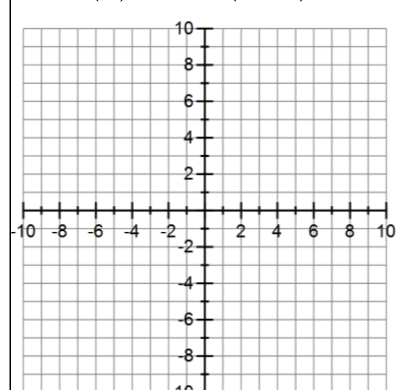
Domain:

Asymptotes:

Key points and transformations:

Range:

d) $f(x) = 2\log(x-3)$



Domain:

Asymptotes:

Key points and transformations:

Range:

D. Finding the inverse of a logarithmic function.

- **$\log_2 x$ means "the exponent to which we raise 2 to get *x*."**
  Pronounced "the logarithm, base 2, of *x*" or "log, base 2, of *x*"

★LOGARITHMS ARE EXPONENTS!★

- ***Logarithm:*** $\log_b a$ means the ***exponent*** to which we raise b to get a.
  b is called the ***base*** of the logarithm (the number being raised to the exponent).
  a is called the ***argument*** of the logarithm (the number you get when you raise the base to the exponent).

The ***logarithmic function of base b***, where $b > 0$ and $b \neq 1$ is denoted by $y = \log_b x$ and is defined by

$$y = \log_b x \text{ if and only if } x = b^y.$$

Example: Change each exponential expression to an equivalent expression involving a logarithm.

a) $5^x = 625$ b) $x^3 = 64$ c) $3^2 = x$

Example: Change each logarithmic expression to an equivalent expression involving an exponent.

a) $\log_3 x = 5$ b) $\log_e 5 = x$ c) $\log_m 2 = n$

E. Evaluating Logarithms

- Instead of "$\log_2 8 = x$," think, what power of 2 equals 8? Or 2 to what power equals 8?
  - $2^x = 8$
  - The answer would be 3 because $2^3 = 8$.

Example: Find the exact value of each logarithm without using a calculator.

a) $\log_3 9 = x$ b) $\log_2 32 = x$ c) $\log_6 1 = x$ d) $\log_5 \frac{1}{125} = x$ e) $\log_7 \sqrt{7} = x$

F. Using a calculator to evaluate logarithms

Use a calculator to evaluate each expression. **Do not forget to put your parentheses in the correct place** if you do not use the fraction button. Do not round until the end of the problem. Round your final answer to the nearest ten-thousandths.

a) $\log 5.83$ b) $\log(-23)$

c) $\ln 21.4$ d) $\frac{\ln 6}{2}$

G. Stories

Example: Chemists define the acidity or alkalinity of a substance according to the formula $\mathrm{pH} = -\log \mathrm{H}^+$, where H^+ is the hydrogen ion concentration, measured in moles per liter. Solutions with a pH value of less than 7 are acidic. Solutions with a pH value of greater than 7 are basic. Solutions with a pH of 7 (such as pure water) are neutral.

a) Suppose you test apple juice and find that the hydrogen ion concentration is $\mathrm{H}^+ = 0.0003$. Find the pH value and determine whether the juice is basic or acidic.

b) Suppose you test ammonia and find that the hydrogen ion concentration is $\mathrm{H}^+ = 1.3 \times 10^{-9}$. Find the pH value and determine whether the juice is basic or acidic.