11.4

Date: 5/2/24 **Section:** 11.4

Objective: I can graph logarithms and write the inverse.

A. Inverses of exponential functions.

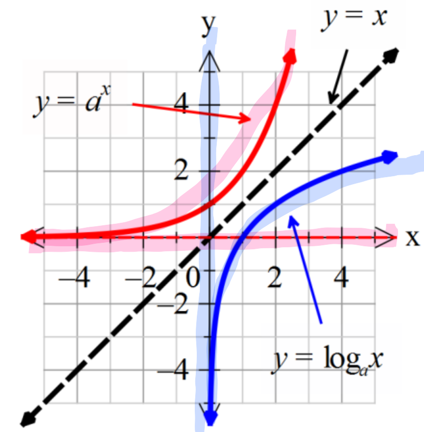
- The logarithmic function $y=\log_a x$ is the inverse of the exponential function $x=a^y$.
- Domain $y=a^x$: $(-\infty,\infty)$ -x-coor. Range $y=a^x$: $(0,\infty)$ -y-coor.
- Domain $y=\log_a x$: $(0,\infty)$. Range $y=\log_a x$: $(-\infty,\infty)$.

Domain of the logarithmic function = ______ of the exponential function $=(0,\infty)$

Range of the logarithmic function = ______ of the exponential function $=(-\infty,\infty)$

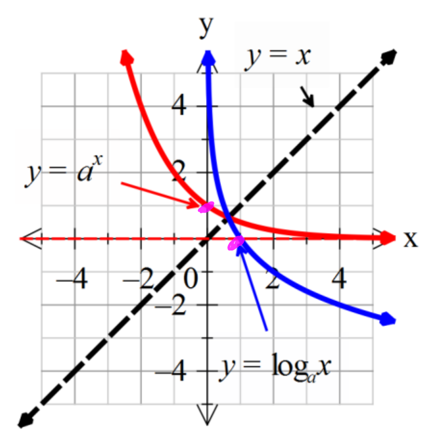
★ **Caution!** You can't take the log of zero or of a negative because it is impossible to get zero or a negative by raising a positive base to an exponent. **The argument of a logarithmic function must be greater than zero.** Restrictions: $\log(x) \neq 0$ or neg, $x>0$

$a>1$



$0 \to \infty$

$0<a<1$

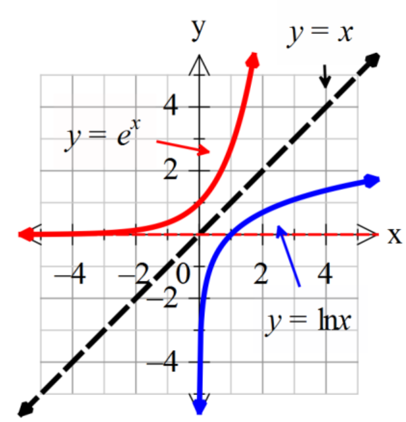


Properties of the Logarithmic Function $f(x)=\log_a x$

- The x-intercept is $(1,0)$. There is no y-intercept.
- The vertical asymptote of the graph is $x=0$.
- The logarithmic function is ______ if $0<a<1$ and ______ if $a>1$. The function is one-to-one.
- Since $y=\log_a x$ is the inverse of $x=a^y$ and the graph $x=a^y$ contains the points $\left(-1,\frac{1}{a}\right)$, $(0,1)$, and $(1,a)$ then the graph of →exp

  $y=\log_a x$ contains the points

  $\left(\frac{1}{a}, -1\right)$, $(1, 0)$, and $(a, 1)$. →log



Common Logarithmic Function: If the base of a logarithmic function is the number 10, then we have the common logarithmic function. If the base a of the logarithmic function is not indicated, it is understood to be 10. That is, $y=\log x$ if and only if $x=10^y$. $\log_{10} x = \log x$

Natural Logarithms: If the base of a logarithmic function is the number e, then we have the natural logarithm function (abbreviated ln). That is, $y=\ln x$ if and only if $x=e^y$. $\log_e x = \ln x$

$y=\ln x$ is the ______ of $y=e^x$

Asymptote Equation: $x=h$ $y=b\log_a(c(x-h))+k$ transformation eq.

$3\sqrt{x+5} = y$

B. Finding the domain of logarithmic functions.

1. $f(x) = \log_2(x+3)$

   $x+3>0$

   $x>-3$

   $(-3, \infty)$

2. $h(x) = -\log_{\frac{1}{2}}(x) + 5$

   $x>0$

   $(0, \infty)$

3. $g(x) = \ln(-x-5) + 3$

   $-x-5>0$

   $-x>5$

   $x<-5$

   $(-\infty, -5)$

C. Graphing logarithmic functions.

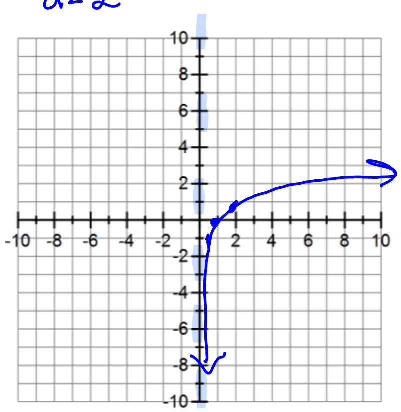
Steps for Graphing Logarithmic Functions:

1. Find the domain
2. Find the asymptotes
3. Graph the asymptotes
4. Find the 3 key points $(1,0)$, $(a,1)$, **and** $\left(\frac{1}{a}, -1\right)$ and apply the appropriate transformations.
5. Plot your points and connect them to form a smooth curve.
6. Find the range

Examples: Graph the following functions.

a) $y = \log_2 x$ $h=0$

$a=2$



Domain: $(0, \infty)$

Asymptotes: $x=0$

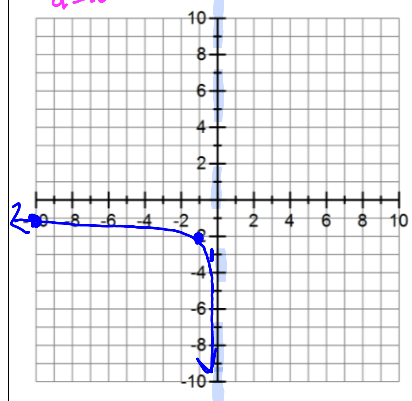
Key points and transformations:

x	y
$\frac{1}{2}$	-1
1	0
2	1

Range: $(-\infty, \infty)$

b) $y = \log(-x) - 2$ $h=0$

$a=10$



Domain: $(-\infty, 0)$

$-x>0$

$x<0$

Asymptotes: $x=0$

Key points and transformations:

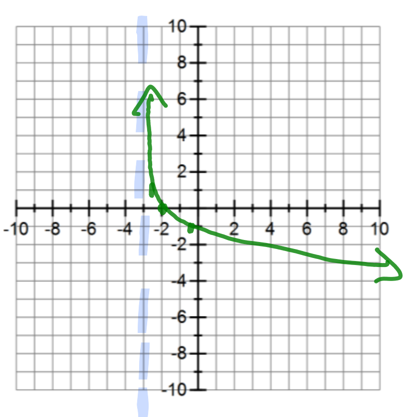
x	y
$\frac{1}{10}$	-1
1	0
10	1

$\rightarrow$

$-x$	$y-2$
$-\frac{1}{10}$	-3
-1	-2
-10	-1

Range: $(-\infty, \infty)$

c) $f(x) = -\ln(x+3)$ $a=e$



Domain: $(-3, \infty)$

$x+3>0$

$x>-3$

Asymptotes: $x=-3$

Key points and transformations:

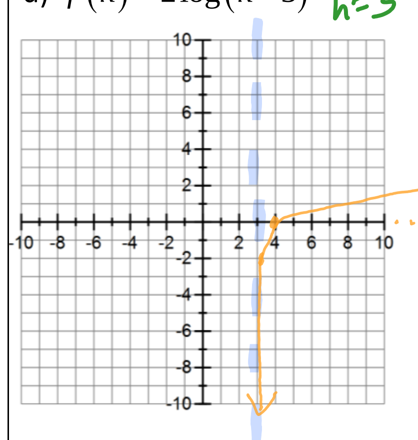
x	y
$\frac{1}{e}$	-1
1	0
e	1

$\rightarrow$

$x-3$	$-y$
$-2.6 \approx \frac{1}{e}-3$	1
-2	0
$-.3 \approx e-3$	-1

Range: $(-\infty, \infty)$

d) $f(x) = 2\log(x-3)$ $h=3$



Domain: $(3, \infty)$

$x-3>0$

$x>3$

Asymptotes: $x=3$

Key points and transformations:

x	y
$\frac{1}{10}$	-1
1	0
10	1

$\rightarrow$

$x+3$	$2y$
$3\frac{1}{10}$	-2
4	0
13	2

Range: $(-\infty, \infty)$

D. Finding the inverse of a logarithmic function.

- $\log_2 x$ **means "the exponent to which we raise 2 to get *x*."**
  Pronounced "the logarithm, base 2, of *x*" or "log, base 2, of *x*"

★LOGARITHMS ARE EXPONENTS!★

- ***Logarithm:*** $\log_b a$ means the ***exponent*** to which we raise b to get a.
  b is called the ***base*** of the logarithm (the number being raised to the exponent).
  a is called the ***argument*** of the logarithm (the number you get when you raise the base to the exponent).

The ***logarithmic function of base b***, where $b > 0$ and $b \neq 1$ is denoted by $y = \log_b x$ and is defined by

$$y = \log_b x \text{ if and only if } x = b^y.$$

$b^x = y$ $\quad \log_b y = x$ $\quad y = b^x$ $\quad b^y = x$ $\quad \log_b x = y$

Example: Change each exponential expression to an equivalent expression involving a logarithm.

a) $5^x = 625$ $\quad \log_5 625$ $\quad \log_5 5^4$ $\quad x = 4$ $\quad x = \log_5 625$

b) $x^3 = 64$ $\quad \log_x$ $\quad 3 = \log_x 64$

c) $3^2 = x$ $\quad \log_3 x = 2$

$x^2 + 3 = y$
$y^2 + 3 = x$
$y^2 = x - 3$
$y = \pm\sqrt{x-3}$

Example: Change each logarithmic expression to an equivalent expression involving an exponent.

a) $\log_3 x = 5$ $\quad$ inverse eq. $\quad x = 3^5$

b) $\log_e 5 = x$ $\quad$ inverse eq. $\quad 5 = e^x$

c) $\log_m 2 = n$ $\quad m^n = 2$

E. Evaluating Logarithms

- Instead of "$\log_2 8 = x$," think, what power of 2 equals 8? Or 2 to what power equals 8?
  - $2^x = 8$
  - The answer would be 3 because $2^3 = 8$.

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Example: Find the exact value of each logarithm without using a calculator.

a) $\log_3 9 = x$ $\quad x = 2$ $\quad \log_3 3^2 = x$

b) $\log_2 32 = x$ $\quad 2^x = 32$ $\quad$ or $\log_2 2^5 = x$ $\quad x = 5$

c) $\log_6 1 = x$ $\quad 6^0 = 1$ $\quad x = 0$

d) $\log_5 \dfrac{1}{125} = x$ $\quad 5^x = \frac{1}{125}$ OR $\log_5 5^{-3} = x$ $\quad x = -3$

e) $\log_7 \sqrt[2]{7} = x$ $\quad \log_7 7^{\frac{1}{2}} = x$ $\quad x = \frac{1}{2}$

F. Using a calculator to evaluate logarithms

Use a calculator to evaluate each expression. **Do not forget to put your parentheses in the correct place** if you do not use the fraction button. Do not round until the end of the problem. Round your final answer to the nearest ten-thousandths.

a) $\log 5.83 \approx .7657$

b) $\log(-23) =$ no sol

c) $\ln 21.4 \approx 3.0634$

d) $\dfrac{\ln 6}{2} \approx .8959$

G. Stories

Example: Chemists define the acidity or alkalinity of a substance according to the formula $pH = -\log H^{+}$, where H^{+} is the hydrogen ion concentration, measured in moles per liter. Solutions with a pH value of less than 7 are acidic. Solutions with a pH value of greater than 7 are basic. Solutions with a pH of 7 (such as pure water) are neutral.

a) Suppose you test apple juice and find that the hydrogen ion concentration is $H^{+} = 0.0003$. Find the pH value and determine whether the juice is basic or acidic.

$pH = -\log .0003$

$pH = 3.5$ acidic

b) Suppose you test ammonia and find that the hydrogen ion concentration is $H^{+} = 1.3 \times 10^{-9}$. Find the pH value and determine whether the juice is basic or acidic.

$-\log(1.3 \times 10^{-9}) \approx 8.9$

basic