11.5

Date: 5/6/24

Objective: I can solve logarithmic equations.

A. Review

Change each logarithmic statement into an equivalent exponential statement.

1. $\log_8 64 = 2$

$8^2 = 64$

2. $\log_2 \frac{1}{16} = -4$

$2^{-4} = \frac{1}{16}$

3. $\log_{10} 8 = x$

$10^x = 8$

4. $\ln x = 5$

$e^5 = x$

Change each exponential statement into an equivalent logarithmic statement.

1. $4^x = 27$

$\log_4 27 = x$

2. $3^{-4} = \frac{1}{81}$

$\log_3 \frac{1}{81} = -4$

3. $9^x = 3.2$

$\log_9 3.2 = x$

Solve the following equation using the laws of exponents.

1. $16^{m+2} = 64$

$(4^2)^{m+2} = 4^3$

$2m+4 = 3$

$2m = -1$

$m = -\frac{1}{2}$

2. $9^{-3n} = 243$

$(3^2)^{-3n} = 3^5$

$-6n = 5$

$n = -\frac{5}{6}$

B. Solving Logarithmic and Exponential Equations

- Use the properties of logarithms and exponents to manipulate the equations.
  a. Remember the *exponential* property: $a^u = a^v \Leftrightarrow u = v$.
- Try rewriting as an *exponential* function: $y = \log_a x \Leftrightarrow x = a^y$ **or** as a *logarithmic* equation: $x = a^y \Leftrightarrow y = \log_a x$

Examples:

a) $\log_{18} 324 = x$

$18^x = 324$

$18^x = 18^2$

$x = 2$

OR

$\cancel{\log_{18}} \cancel{18}^2 = x$

$x = 2$

b) $6^{x-4} = 11$

$\log_6(11) = x - 4$

$\log_6(11) + 4 = x$

c) $\ln_e e^{2x} = 6$

$e^6 = e^{2x}$ OR $\cancel{\ln_e} \cancel{e}^{2x} = 6$

$6 = 2x$ $\quad 2x = 6$

$x = 3$ $\quad x = 3$

d) $\frac{3 \cdot (10)^{3-x}}{3} = \frac{7}{3}$

$10^{3-x} = \frac{7}{3}$

$\log_{10} \frac{7}{3} = 3 - x$

$\log\left(\frac{7}{3}\right) - 3 = -x$

$-\log\left(\frac{7}{3}\right) + 3 = x$

e) $\log_3(3x-1) = 2$

$3^2 = 3x - 1$

~~$9 = 3x - 1$~~

$10 = 3x$

$x = \frac{10}{3}$

f) $2^{-x} = 1.5$

$\log_2 1.5 = -x$

$-\log_2 1.5 = x$

g) $\log_6 216 = 3x + 2$

$\log_6(216) - 2 = 3x$

$\frac{\log_6(216) - 2}{3} = x$

$\frac{\cancel{\log_6} \cancel{6}^3 - 2}{3} = x$

$\frac{3-2}{3} = x = \frac{1}{3}$

OR $\cancel{\log_6} \cancel{6}^3 = 3x + 2$

$3 = 3x + 2$

$1 = 3x$

$\frac{1}{3} = x$

OR $6^{3x+2} = 216$

$6^{3x+2} = 6^3$

$3x + 2 = 3$

$3x = 1$

$x = \frac{1}{3}$

h) $e^{4x+3} = 9$

$\log_e 9 = 4x + 3$ or $\ln 9 = 4x + 3$

$\log_e(9) - 3 = 4x$ $\quad \ln(9) - 3 = 4x$

$\frac{\log_e(9) - 3}{4} = x$ $\quad \frac{\ln(9) - 3}{4} = x$

C. Present Value is when we know the interest rate and how much money we want to end with, but we don't know how much we should invest to start. For this situation, we will use the same interest formulas, but then you will need to solve for the variable instead of using your calculator to evaluate.

Compound Interest Formula

The amount A after t years due to a principal P invested at an annual interest rate t compounded n times per year is $\boldsymbol{A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}}$.

Continuous Compounding

The amount A after t years due to a principal P invested at an annual interest rate r compounded continuously is $\boldsymbol{A = Pe^{rt}}$.

Example: How much money must be invested now in order to end up with \$20,000 in 10 years at
a) 5% compounded quarterly? b) 3.8% compounded continuously?

Example: A zero-coupon (noninterest-bearing) bond can be redeemed in 9 years for \$1200. How much should you be willing to pay for it now if you want a return of
a) 7% compounded monthly? b) 6% compounded continuously?

Example: What annual rate of interest compounded annually should you seek if you want to double your investment in 7 years?