11.6

Date: 5/8/24

Objective: I can use logarithm properties.

Properties of exponents	Properties of logarithms
1. $a^0 = 1$	1. $\log_a 1 = 0$
2. $a^1 = a$	2. $\log_a a = 1$
3. $a^{\log_a x} = x$	3. $\log_a a^x = x$
4. $(ab)^m = a^m b^m$	4. $\log_x (ab)^m = \log_x a^m b^m$
5. $a^m = a^n$ $m = n$	5. $\log_a m = \log_a n$ $m = n$
6. $x^m \cdot x^n = x^{m+n}$	6. $\log_a m + \log_a n = \log_a mn$
7. $\frac{x^m}{x^n} = x^{m-n}$	7. $\log_a m - \log_a n = \log_a \frac{m}{n}$
8. $(x^m)^n = x^{mn}$	8. $\log_a r^m = m \log_a r$

Example:

Simplfiy. Do not use a calculator.

1. $\log_4 1 = 0$

2. $5^{\log_5 3} = 3$

3. $\log_7 7^{-3} = -3$

4. $\ln e^5 = 5$

5. $\log_2 64$

$\cancel{\log_2} 2^6 = 6$

6. $\log_7 \frac{1}{49}$

$\cancel{\log_7} 7^{-2} = -2$

7. $\log \sqrt[2]{10^1}$

$\cancel{\log_{10}} 10^{\frac{1}{2}} = \frac{1}{2}$

Example:

Condense to one logarithm or simplify.

$x^8 \cdot x^4$

1. $\ln 8 + \ln x$

$\ln 8x$

2. $\text{Log}\, u - \log v$

$\log \frac{u}{v}$

3. $\frac{1}{4} \log_6 x$

$\log_6 x^{\frac{1}{4}}$

4. $3 \log_7 x + \log_7 4$

$\log_7 x^3 + \log_7 4$

$\log_7 (4x^3)$

5. $5 \log 2 - \log 5x$

$\log 2^5 - \log 5x$

$\log \frac{2^5}{5x}$

6. $4 \ln(uv) - 3 \ln(vw)$

$\ln u^4 v^4 - \ln v^3 w^3$

$\ln \frac{u^4 v^4}{v^3 w^3}$

$\ln \frac{u^4 v}{w^3}$

7. $\log(x-4) + \log(6x+5)$

$(x-4)(6x+5)$

$\log(6x^2 - 19x - 20)$

Example:

Expand or write with more than one logarithm.

1. $\log 5x$

$\log 5 + \log x$

2. $\log \frac{5}{x}$

$\log 5 - \log x$

3. $\log_7 x^5$

$5 \log_7 x$

4. $\ln(x^2 e^x)$

$\ln x^2 + \ln e^x$

$2\ln(x) + x$

5. $\log_6 \frac{\sqrt[4]{y}}{\sqrt[4]{x}}$

$\log_6 \frac{y^{\frac{1}{4}}}{x^{\frac{1}{4}}}$

$\log_6 y^{\frac{1}{4}} - \log_6 x^{\frac{1}{4}}$

$\frac{1}{4}\log_6 y - \frac{1}{4}\log_6 x$

Change of base formula:

used to type in calculator if not base 10 or e

$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$

Example:

Write each logarithm using change of base formula. Then evaluate the logarithm rounding to the nearest ten-thousandth. Use common logarithms.

1. $\log_6 9 = \frac{\log 9}{\log 6} \approx 1.2263$

2. $\log_6 3.5$

$\frac{\log 3.5}{\log 6} \approx .6992$

Example:

Write each logarithm using change of base formula. Then evaluate the logarithm rounding to the nearest ten-thousandth. Use natural logarithms.

1. $\log_3 8$ $\frac{\ln 8}{\ln 3} \approx 1.8928$

2. $\log_2 4.12$

Sometimes, when we substitute the given information into the equation the variable is in the exponent. When this happens, you need to use logarithms to solve for the variable.

Example:

1. The population of a bacteria can be modeled by $P(t) = \boxed{350}(1.25)^t$. According to this model, when will the population be 800? Round to the nearest tenth of a day.

t

$$800 = 350(1.25)^t$$

$$\frac{800}{350} = 1.25^t$$

$$\log_{1.25} \frac{800}{350} = t$$

$$\frac{\log \frac{800}{350}}{\log 1.25} = t$$

$t \approx 3.7$ days

2. The value of a cell phone that is t years old can be modeled by $v(t) = 1{,}399(0.18)^t$. According to the model, when will the phone be worth \$300? Round to the nearest tenth of a year.

$$300 = 1399(.18)^t$$

$$\frac{300}{1399} = .18^t$$

$$\log_{.18} \frac{300}{1399} = t$$

$t \approx .9$ yrs.