11.7

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Objective: I can solve logarithmic equations and find the domain.

A. Review

1) $\log_3 x = 4$

2) $27\left(\frac{1}{3}\right)^{x/5} = 3$

$\frac{3}{27}$

$\left(\frac{1}{3}\right)^{\frac{x}{5}} = \frac{1}{9}$

$x = 10$

$3^{-\frac{x}{5}} = 3^{-2}$

$\left(\frac{-x}{5} = -2\right)-5$

3) $16 \cdot 4^{x/3} = 1024$

B. Use the Properties of Logarithms and Exponents to solve equations.

- To avoid extraneous solutions, determine the domain of the variable first.
- Use the *properties of logarithms and exponents* to manipulate the equations.
- Try rewriting as an exponential or logarithmic function: $y = \log_a x \Leftrightarrow x = a^y$
- Remember the properties: $\log_a M = \log_a N \Leftrightarrow M = N$ and $a^u = a^v \Leftrightarrow u = v$ (Make the bases the same).
- Check your solution by substituting into the original equation.

a) $2.05^x = 4.36$

$\log_{2.05} 4.36 = x$

$\frac{\log 4.36}{\log 2.05} = x$

$x \approx 2.0513$

b) $30e^{0.014x} = 600$

$\ln e^{.014x} = \ln 20$

$\frac{.014x}{.014} = \frac{\ln 20}{.014}$

$x \approx 213.9809$

c) $8 - 5e^{-x} = -12$

d) $2^{4-x} - 7 = 14$

$2^{4-x} = 21$

$\log_2 21 = 4 - x$

$\log_2(21) - 4 = -x$

$-\log_2(21) + 4 = x$

$\frac{-\log 21}{\log 2} + 4 = x$

$x \approx -.3923$

e) $\ln x = 8$

f) $-4\log(x+5) - 3 = -4$

$x+5>0$

$x>-5$

$-4\log(x+5) = -1$

$\log_{10}(x+5) = \frac{1}{4}$

$10^{\frac{1}{4}} = x+5$

$10^{\frac{1}{4}} - 5 = x$

$x \approx -3.2217$

g) $\cancel{\log x = \log_4 9}$

h) $\log_4 x = \log_4 (3x-8)$

$x>0$

$3x-8>0$

$x > \frac{8}{3} = 2\frac{2}{3}$

$x = 3x-8$

$-2x = -8$

$x = 4$

i) $\log_3(2x+5) + \log_3 27 = 5$

$\log_3 3^3$

$2x+5>0$

$x > \frac{-5}{2}$ ☆

$\log_3 27(2x+5) = 5$ OR $\log_3(2x+5) + 3 = 5$

$\log_3(2x+5) = 2$

$3^2 = 2x+5$

$4 = 2x$

$x = 2$ ☆

j) $3\log_2(x-1) + \log_2 4 = 5$

k) $3\log_2(x-1) + \log_2 4 = 5$

l) $\ln(5x) - \ln(10) = 5$

$5x>0$

$x>0$

$\ln\left(\frac{5x}{10}\right) = 5$

$2\left(e^5 = \frac{x}{2}\right)$

$x \approx 296.8263$

m) $\log_6(x+4) + \log_6(x+3) = 1$

$x>-4$

$x>-3$

$\log_6(x^2+7x+12) = 1$

$6^1 = x^2+7x+12$

$x^2+7x+6=0$

$(x+6)(x+1)=0$

$x = \cancel{-6}, -1$

n) $\log_2 x + \log_2(x-2) = \log_2(x+4)$

$x>0$

$x>2$

$x>-4$

$\log_2(x^2-2x) = \log_2(x+4)$

$x^2-2x = x+4$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x = 4, \cancel{-1}$

C. Present Value is when we know the interest rate and how much money we want to end with, but we don't know how much we should invest to start. For this situation, we will use the same interest formulas, but then you will need to solve for the variable instead of using your calculator to evaluate.

Compound Interest Formula

The amount A after t years due to a principal P invested at an annual interest rate t compounded n times per year is $\boldsymbol{A = P \cdot \left(1+\frac{r}{n}\right)^{nt}}$.

Continuous Compounding

The amount A after t years due to a principal P invested at an annual interest rate r compounded continuously is $\boldsymbol{A = Pe^{rt}}$.

Example: How much money must be invested now in order to end up with $20,000 in 10 years at

a) 5% compounded quarterly?

b) 3.8% compounded continuously?

Example: What annual rate of interest compounded annually should you seek if you want to double your investment in 7 years?

Exponential Growth and Decay Models

***Law of Uninhibited Growth or Decay*:**

Many natural phenomena have been found to follow the law that an amount A varies with time t according to the function $A(t) = A_0 e^{kt}$, where A_0 is the original amount at time $t = 0$ and k is a constant of growth or decay (growth if $k > 0$, decay if $k < 0$.)

rate (%) as decimal

Example: The number N of bacteria present in a culture at time t hours obeys the law of uninhibited growth where $N(t) = 1000e^{\boxed{0.01}t}$.

a) Determine the number of bacteria at $t = 0$ hours.

1000 bacteria

b) What is the growth rate of the bacteria? $\boxed{1\%}$

$.01$

c) What will the population be after 4 hours? $\leftarrow t$

$1000e^{.01(4)} \approx 1040$ bacteria

d) When will the number of bacteria reach 1700? $N(t)$

$1700 = 1000e^{.01t}$

$1.7 = e^{.01t}$

$\frac{\ln 1.7 = .01t}{.01}$

$t \approx 53.1$ hours

e) When will the number of bacteria double?

$2000 = 1000e^{.01t}$

$2 = e^{.01t}$

$\frac{\ln 2 = .01t}{.01}$

$t \approx 69.3$ hrs.

Example: Iodine 131 is a radioactive material that decays according to the function $A(t) = A_0e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine 131.

A_0

a) What is the decay rate of iodine 131? -8.7%

b) How much iodine 131 is left after 9 days? $100e^{-.087(9)} \approx 45.7$ grams

c) When will 70 grams of iodine 131 be left?

$\frac{70 = 100e^{-.087t}}{100}$

$.7 = e^{-.087t}$

$\frac{\ln .7 = -.087t}{-.087}$

$t \approx 4.1$ days

d) What is the half-life of iodine 131? (when $A = \frac{1}{2}A_0$.)

$50 = 100e^{-.087t}$

$.5 = e^{-.087t}$

$\frac{\ln .5 = -.087t}{-.087}$

$t \approx 8.0$ days