4.1

Date: 10/18/23 Section:

Objective: I can find the zeros and x-intercepts of a functions.

Vocabulary

Zeros: where it touches all solutions including imaginary

x-intercepts: where it crosses x-axis

Fundamental Theorem of Algebra: degree = # of zeros

EXAMPLE: How many zeros does each polynomial have?

a. $f(x) = x^6 + 3x^4 - 6x^3 + 8$ standard form 6

b. $f(x) = (x+4)(x-3)(2x+5)$ factored form 3

c. $f(x) = 2x(x-1)(x^2-9)$ $2x(x-1)(x+3)(x-3)$ 4

Ways to find the zeros (solve for the variable)

Way 1—only one variable in the equation:	Way 2—more than one variable in the equation:	Way 3—polynomial is prime (doesn't factor):
1. Isolate the variable or the parenthesis 2. Do the inverse of the variable or the parenthesis 3. Solve for the variable **EXAMPLE: Find the zeros.** $4\boxed{(x-3)^2} + 5 = 41$ $4(x-3)^2 = 36$ $\sqrt{(x-3)^2} = \sqrt{9}$ $x-3 = \pm 3$ $x = 6, 0$	1. Set equation equal to 0 2. FACTOR 3. Set each factor equal to 0 4. Solve for x **EXAMPLE: Find the zeros.** $x^2 + 4x = 12$ $x^2 + 4x - 12 = 0$ -12, -2, 6, 4 $(x-2)(x+6) = 0$ $x = 2, -6$	1. Set equation equal to 0 2. Find the values of $a, b,$ and c 3. Substitute those values into the Quadratic Formula $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ 4. Use calculator to evaluate what is under the square root sign (DO NOT use the square root sign!) 5. Simplify the square root 6. Simplify the numbers outside the square root **EXAMPLE: Find the zeros.** $y = x^2 + x - 1$ $ax^2 + bx + c$ $a=1$, $b=1$, $c=-1$ $x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$ $x = \frac{-1 \pm \sqrt{5}}{2}$

EXAMPLE: Find the zeros using two different methods above.

$y = 2x^2 + 5x - 3$ factor / QF

way 2 - factor

-6, 6, 5, -1

$2x^2 + 6x - x - 3 = 0$

$2x(x+3) - 1(x+3) = 0$

$(2x-1)(x+3) = 0$

$x = \frac{1}{2}, -3$

way 3 - QF

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-3)}}{2(2)}$

$x = \frac{-5 \pm \sqrt{49}}{4}$

$x = \frac{-5 \pm 7}{4}$ $\frac{-12}{4}$ $\frac{2}{4}$ $\boxed{\frac{1}{2}, -3}$

EXAMPLE: Find the zeros

1. $f(x) = x^2 + 8x + 10$
2. $f(x) = 5x^2 - 20x - 9$
3. $f(x) = 3x^2 - 14x - 5$

Determining End Behavior: what happens at the "ends" of the graph. We write end behavior using limits.

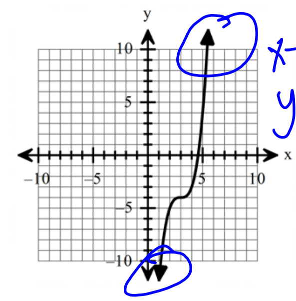
- **From a graph:** This means we determine what is happening to the function as on the right end of the x-axis and what is happening to the function on the left end of the x-axis.

EXAMPLE: Left End Behavior $\lim_{x\to-\infty} f(x) = -\infty$

As x goes left to $-\infty$, the function goes down to $-\infty$

Right End Behavior $\lim_{x\to\infty} f(x) = \infty$

As x goes right to ∞, the function goes up to ∞



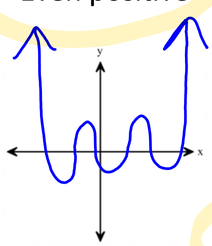
$x \to \infty$
$y \to \infty$

$x \to -\infty$
$y \to -\infty$

- **From an equation:** To find the end behavior of a function, we look at two things.
  1. the leading coefficient: is it a positive or negative number?
  2. the degree of the polynomial: is it an even or odd number?

Steps for finding the end behavior

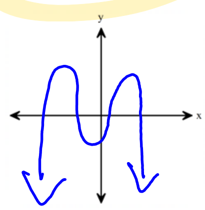
Even positive



Ex. x^6

$\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to+\infty} f(x) = \infty$

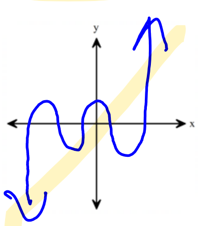
Even negative



Ex. $-x^4$

$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to+\infty} f(x) = -\infty$

Odd positive

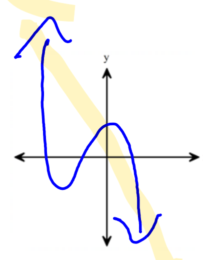


Ex. x^5

$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to+\infty} f(x) = \infty$

left right

Odd negative

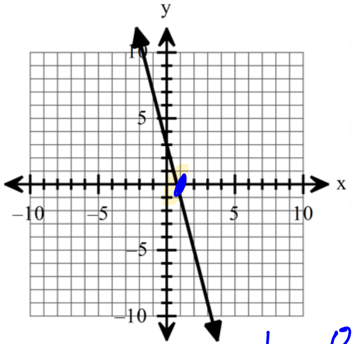


Ex. $-x^3$

$\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to+\infty} f(x) = -\infty$

Find the end behavior of each function given. Write the end behavior in limit notation if it is not written for you.

1.



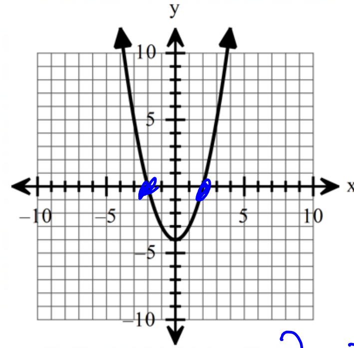
odd neg

Number of Zeros: 1

Left End Behavior $\lim_{x\to-\infty} f(x) = \infty$

Right End Behavior $\lim_{x\to\infty} f(x) = -\infty$

2.



even pos

Number of Zeros: 2

Left End Behavior $\lim_{x\to-\infty} f(x) = \infty$

Right End Behavior $\lim_{x\to\infty} f(x) = \infty$

3. $\boldsymbol{f(x) = -3x^6 + 5x^4 - x^2 + 3}$

Number of Zeros: 6

Degree: 6 L.Coefficent: neg

Left End Behavior $\lim_{x\to-\infty} f(x) = -\infty$

Right End Behavior $\lim_{x\to\infty} f(x) = -\infty$

4. $\boldsymbol{g(x) = 2x^3 + x^2 - x + 1}$

Number of Zeros: 3

Degree: 3 L.Coefficent: pos

Left End Behavior $\lim_{x\to-\infty} f(x) = -\infty$

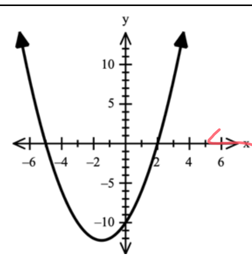
Right End Behavior $\lim_{x\to\infty} f(x) = \infty$

Match the graph with its equation (draw a line connecting them) by Identifying the end behavior.

1.

Even or Odd?

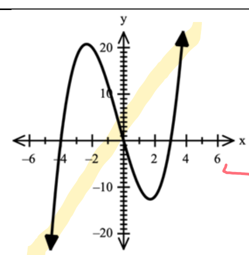
Positive or Negative?



2.

Even or Odd?

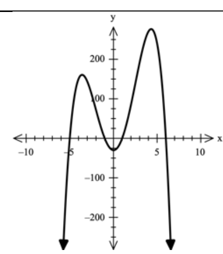
Positive or Negative?



3.

Even or Odd?

Positive or Negative?



A. $f(x) = -x^4 + x^3 + 31x^2 - x - 30$

Number of Zeros: 4

Even or Odd? + or − ?

$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to+\infty} f(x) = -\infty$

B. $f(x) = x^2 + 3x - 10$

Number of Zeros: 2

Even or Odd? + or − ?

$\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to+\infty} f(x) = \infty$

C. $f(x) = x^3 + x^2 - 12x$

Number of Zeros: 3

Even or Odd? + or − ?

$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to+\infty} f(x) = \infty$

Finding & graphing zeros of the function: In order to find the zeros of a function we must **factor** the polynomial COMPLETELY! Then we will set each factor equal to zero, and solve for *x*.

For each problem, find the zeros and graph them. Identify the end behavior and draw it on the graph.

1. $f(x) = 3(x-6)(x+6)$

$x = 6, -6$

End Behavior

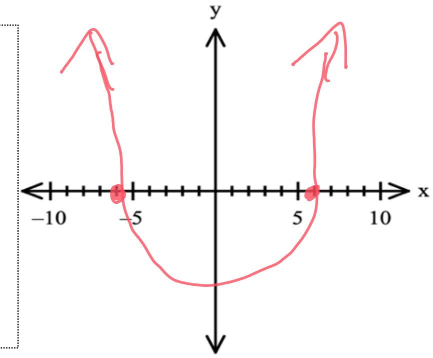
Degree: 2 — *Even or Odd?* Even

Leading Coefficient: 3 — *Positive or negative?* Positive

Which tells us? goes to heaven

$\lim_{x\to-\infty} f(x) = \infty$ $\quad\lim_{x\to+\infty} f(x) = \infty$

Zeros: ______________



2. $f(x) = x(x+5)(x-8)$

End Behavior

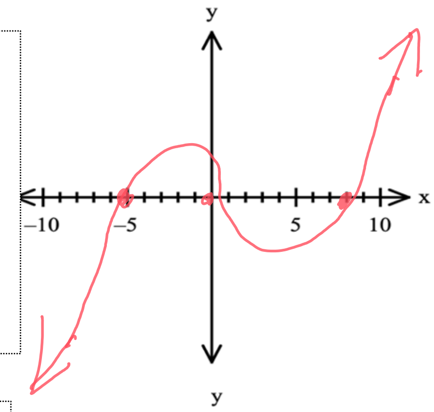
Degree: 3 — *Even or Odd?* Odd

Leading Coefficient: 1 — *Positive or negative?* Positive

Which tells us? pos against neg

$\lim_{x\to-\infty} f(x) = -\infty$ $\quad\lim_{x\to+\infty} f(x) = \infty$

Zeros: 0, −5, 8



3. $f(x) = -x^2 - 4x + 5$

−5
+5 −1
+4

$-(x^2+4x-5)=0$

$-(x+5)(x-1)=0$

End Behavior

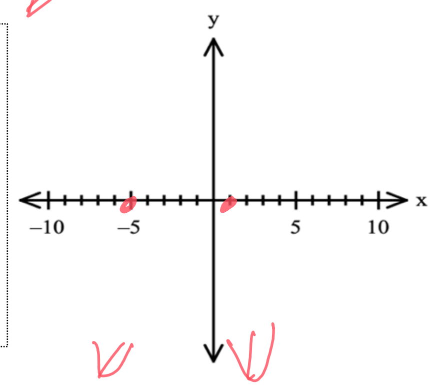
Degree: 2 — *Even or Odd?* Even

Leading Coefficient: −1 — *Positive or negative?* negative

Which tells us? going down

$\lim_{x\to-\infty} f(x) = -\infty$ $\quad\lim_{x\to+\infty} f(x) = -\infty$

Zeros: −5, 1



4. $f(x) = -2x^3 - 14x^2 - 12x$

$0 = -2x(x^2+7x+6)$

$0 = -2x(x+6)(x+1)$

End Behavior

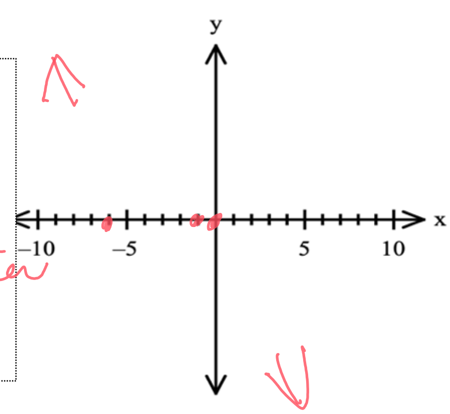
Degree: 3 — *Even or Odd?* Odd

Leading Coefficient: −2 — *Positive or negative?* negative

Which tells us? everybody hates each other

$\lim_{x\to-\infty} f(x) = \infty$ $\quad\lim_{x\to+\infty} f(x) = -\infty$

Zeros: 0, −6, −1



☆ Remember

$1+\sqrt{2}$ is a number you can graph.

change it to a decimal.

$1 + 1.41 = 2.41$ — graph this