

5.4

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Objective: I can solve rational equations.

REVIEW: Simplify the following and fill in the blank:

a) $\frac{10}{0}$ *undef*

b) $\frac{0}{10} = 0$

c) Anything divided by zero is undef.

d) The denominator (bottom) of a fraction **can't** equal 0.

e) Finding Restrictions: If there is a variable in the denominator, set the denominator equal to 0 and solve for the variable. (Do for each denominator that is different). If there is no variable in the denominator, there is no restriction.

EXAMPLES: State the restrictions for each rational equation:

a) $\frac{5}{x+4} = 2$ *$\frac{5}{-4+4} = 2$*

*$x+4 \neq 0$
 $\frac{5}{0} = 2$
 $x \neq -4$*

b) $\frac{4x}{8x-3} = \frac{7}{x}$

*$8x-3 \neq 0$ $x \neq 0$
 $8x \neq 3$
 $x \neq \frac{3}{8}$*

c) $\frac{5x-2}{5} = \frac{2x}{5}$

*no restrictions
because no var.
in denom
 $5 \neq 0$*

d) $\frac{x-2}{x^2} = \frac{1}{2x}$

*$x^2 \neq 0$ $2x \neq 0$
 $x \neq 0$*

e) $\frac{x+9}{x^2+6x+8} = \frac{4x+1}{x-6}$

*$(x+4)(x+2)$
 $x+4 \neq 0$ $x+2 \neq 0$ $x-6 \neq 0$
 $x = -4, -2, 6$*

f) $\frac{7x+4}{x^2+3x} = \frac{1}{x}$

*$x(x+3)$
 $x \neq -3, 0$*

Steps for Solving Rational Equations:

same as
+/-
solve

1. Factor the denominator and find Lowest Common Denominator (LCD).
2. Multiply top and bottom of each fraction to get common denominator.
3. Multiply the entire equation by the LCD to get rid of the fractions.
4. Solve for the variable.
5. State the restrictions and check against your answers.

EXAMPLES: State the restrictions. Solve the equation algebraically. Identify the extraneous solutions.
Show work!

$$1. \frac{5x-1}{5x} - \frac{1}{5x} = \frac{4}{5x}$$

$$\left(\frac{5x}{5x} + \frac{-1}{5x} = \frac{4}{5x} \right) \frac{5x}{1}$$

$$5x - 1 = 4$$

$$5x = 5$$

$$x = 1$$

$5x \neq 0$
 $x \neq 0$

$$2. \frac{4}{n-1} + \frac{6}{n-1} = \frac{1}{n-1}$$

$$\left(\frac{4n-4}{n-1} + \frac{6}{n-1} = \frac{1}{n-1} \right) (n-1)$$

$$4n-4+6=1$$

$$4n+2=1$$

$$4n=-1$$

$$n = -\frac{1}{4}$$

$n-1 \neq 0$
 $n \neq 1$

$$3. \frac{x-1}{x(x+3)} + \frac{x+2}{x(x+3)} = \frac{1}{x(x+3)}$$

$$\left(\frac{x}{x(x+3)} + \frac{x+2}{x(x+3)} = \frac{x+3}{x(x+3)} \right) (x)(x+3)$$

$$x + x + 2 = x + 3$$

$$2x + 2 = x + 3$$

$$-x - 2 = -x - 2$$

$$x = 1$$

$x(x+3) \neq 0$
 $x \neq -3, 0$

$$4. \frac{-5}{x^2-7x+12} = \frac{-5(x-3)}{3-x}$$

$-(x-4)(x-3)$ $-(x-3)$ $-(x-4)(x-3)$

$$5. \frac{2}{x-3} + \frac{6}{x+5} = \frac{2}{x^2+2x-15}$$

$(x+1)(x-4)$

$$6. \frac{(x+1)(x-4)}{(x+1)x} - \frac{3x}{x(x+1)} + \frac{4}{x^2+x} = 0$$

$$\left(\frac{-5}{-(x-4)(x-3)} + \frac{-2x+8}{-(x-3)(x-4)} = \frac{-5x+15}{-(x-4)(x-3)} \right) \frac{-(x-4)(x-3)}{1}$$

$$-5 - 2x + 8 = -5x + 15$$

$$-2x + 3 = -5x + 15$$

$$+5x - 3 = +5x - 15$$

$$3x = 12$$

$$x = 4$$

$-(x-4)(x-3) \neq 0$
 $x \neq 4, 3$

no solution
 $x=4$ is extraneous

$$x^2 - 4x + x - 4 = 0$$

$$x(x+1) \left(\frac{x^2-3x-4}{x(x+1)} + \frac{-3x}{x(x+1)} + \frac{4}{x(x+1)} = 0 \right)$$

$$x^2 - 3x - 4 - 3x + 4 = 0$$

$$x^2 - 6x = 0$$

$$x(x-6) = 0$$

$$x = 0, 6$$

$x(x+1) \neq 0$
 $x \neq 0, -1$