

6.2

Date: 12/8/23

Objective: I can find parts of a rational graph from an equation.

Task – Winner, Winner

1. Imagine you won the lottery and were given a big pot of money. Of course, you would want to share with your friends and family. If you split the money evenly between yourself and one friend, what would be each person's share of the money?

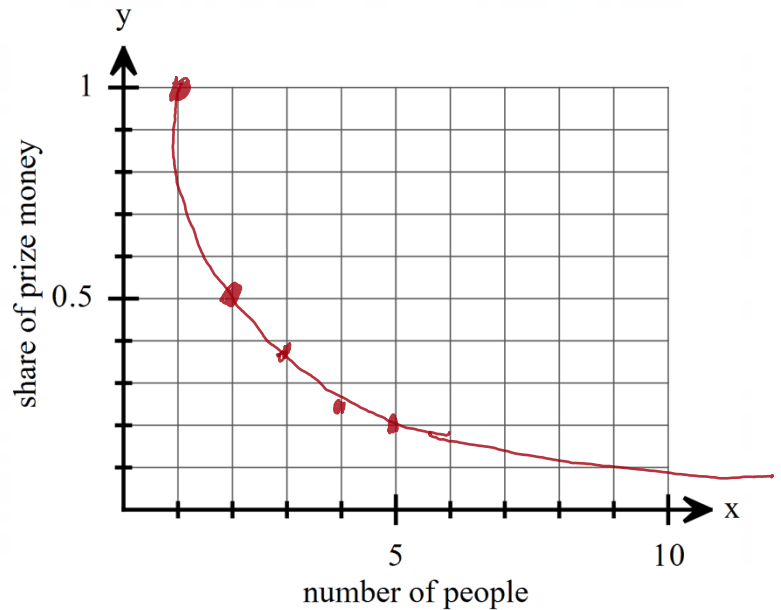
$\frac{1}{2}$ 50%

2. If three people shared the prize money, what would be each person's share of the money?

$\frac{1}{3}$

3. Model the situation with a table, equation, and graph.

x	y
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
4	$\frac{1}{4}$
5	$\frac{1}{5}$



What is the relationship between the number of people and the share of money?

Equation: $\frac{1}{p} = y$ $\frac{1}{x} = y$

4. As the number of friends gets bigger, what do you think happens to the share of the prize money?

If you shared with 1,000 friends, how much would each person's share of the prize money? 10,000 friends? 1,000,000 friends?

$\frac{1}{10000}$

$\frac{1}{1000000}$

$\frac{1}{1000}$

5. Use mathematical notation to describe the behavior of this function as $x \rightarrow \infty$.

$$\lim_{x \rightarrow \infty} f(x) = 0$$

Next, let's look at the interval (0,1]

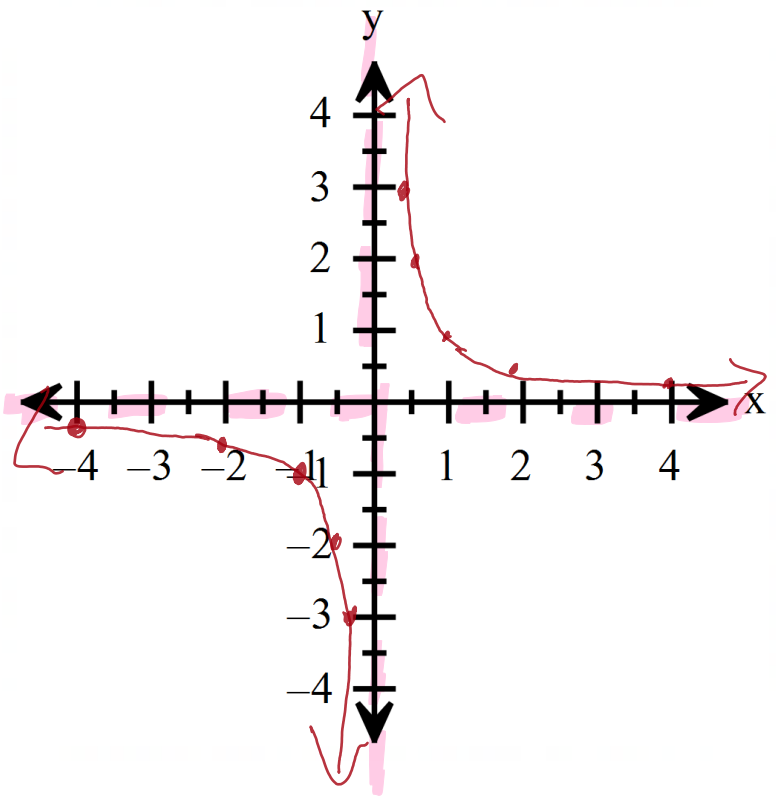
6. Can you have 0 people that can share the prize money? *no*

7. Will you ever get to 0 shares of the prize money? *no*

8. What would this look like on a graph? *asymptotes*

9. Complete the table using the given values for x and graph the function $f(x) = \frac{1}{x}$

x	y
-4	$-\frac{1}{4}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$-\frac{1}{3}$	-3
0	<i>undef</i>
$\frac{1}{3}$	3
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$
4	$\frac{1}{4}$



10. Compare the table & graph of $f(x) = \frac{1}{x}$ to answer the following questions.

a. How do the y-values when x is positive compare to the y-values when x is negative?

reciprocal

b. Where does the graph not exist? Why?

0 can't have 0 shares

c. What is this called?

asymptote

d. What is the domain? How does part b,c affect the domain?

$(-\infty, 0) \cup (0, \infty)$

****This type of graph is a rational graph because there is a variable in the denominator.****

Finding intercepts algebraically

Just like all of the parent functions we have done so far, you can find the intercepts from the equation.

- To find the **x-intercept**, make the **y-value 0** and solve for **x**.
- To find the **y-intercept**, make the **x-value 0** and evaluate to find the **y-value**.

EXAMPLES:

1. $f(x) = \frac{1}{x+5}$

x-intercept(s): none

y-intercept: $(0, \frac{1}{5})$

$$(x+5) \left(0 = \frac{1}{x+5} \right)$$

$$\frac{1}{0+5}$$

$$0 = 1$$

2. $f(x) = \frac{x+4}{x-1}$

x-intercept(s): $(-4, 0)$

y-intercept: $(0, -4)$

$$(x-1) \left(0 = \frac{x+4}{x-1} \right)$$

$$\frac{0+4}{0-1}$$

$$0 = x+4$$

3. $f(x) = \frac{(x-4)(x+3)}{(x-1)(x+1)}$

x-intercept(s): $(4, 0)$ $(-3, 0)$

y-intercept: $(0, 12)$

$$(x-1)(x+1) \left(0 = \frac{(x-4)(x+3)}{(x-1)(x+1)} \right)$$

$$\frac{(0-4)(0+3)}{(0-1)(0+1)}$$

$$0 = (x-4)(x+3)$$

$$x-4=0 \quad x+3=0$$

$$\frac{-12}{-1}$$

Finding Vertical Asymptotes & Domain algebraically

You can find the asymptotes without graphing.

- To find the **vertical asymptote(s)**, set the denominator equal to 0 and solve. (Find the restrictions.)

Domain: Find the restrictions (vertical asymptote), then write the domain in interval notation (excluding those restrictions).

EXAMPLES:

1. $f(x) = \frac{1}{x+5}$

Vertical Asymptote(s): $x = -5$

Domain: $(-\infty, -5) \cup (-5, \infty)$

$$x+5 \neq 0$$

2. $f(x) = \frac{x+4}{x-1}$

Vertical Asymptote(s): $x = 1$

Domain: $(-\infty, 1) \cup (1, \infty)$

$$x=1$$

3. $f(x) = \frac{(x-4)(x+3)}{(x-1)(x+1)}$

Vertical Asymptote(s): $x = -1$

Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Finding Asymptotes & Domain algebraically

- To find the **vertical asymptote(s)**, set the denominator equal to 0 and solve. (Find the restrictions.)
- To find the **horizontal asymptote**, you have 2 rules.
 1. If the degree of the numerator is less than the degree of the denominator the horizontal asymptote is $y = 0$
 2. If the degree of the numerator is equal to the degree of the denominator the horizontal asymptote is $y = \frac{\text{leading coefficients}}{\text{leading coefficients}}$

$$\frac{x^2}{x^3}$$

$$\frac{x^2}{x^2}$$

EXAMPLES:

$$1. f(x) = \frac{1}{x-2}$$

Vertical Asymptote(s): $x=2$ Domain: $(-\infty, 2) \cup (2, \infty)$

Horizontal Asymptote: $y=0$

$$\frac{x^0}{x^1}$$

$$2. f(x) = \frac{3x+4}{2x-1}$$

Vertical Asymptote(s): $x = \frac{1}{2}$ Domain: $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

Horizontal Asymptote: $y = \frac{3}{2}$

$$3. f(x) = \frac{2x^2-x-6}{x^2-1}$$

$$\frac{(2x+3)(x-2)}{(x+1)(x-1)}$$

Vertical Asymptote(s): $x=1$ $x=-1$ Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Horizontal Asymptote: $y=2$

$$\begin{array}{r} -12 \\ -4 \times 3 \\ \hline -1 \end{array}$$

EXAMPLES:

$$1. f(x) = \frac{x}{x^2-3x+2}$$

$$\frac{x}{(x-2)(x-1)}$$

x-intercept(s): $(0, 0)$
 $(0 = \frac{x}{x^2-3x+2}) \Rightarrow x^2-3x+2 = 0 \Rightarrow x=0$

Vertical Asymptote(s): $x=2$
 $x=1$

Horizontal Asymptote: $y=0$

y-intercept: $(0, 0)$
 $\frac{0}{0^2-3(0)+2}$

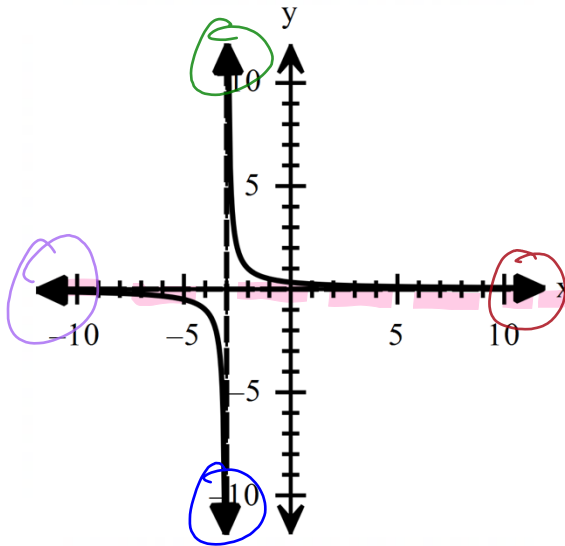
Domain: $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

Not only do rational graphs have end behavior, but they also have limits at the asymptotes.

- The negative symbol exponent means as you approach from left side.
- The positive symbol exponent means as you approach from the right side.

Example: Evaluate the limit and end behavior based on the graph of $f(x)$ shown.

1.



End Behavior:

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

Limits at the asymptote:

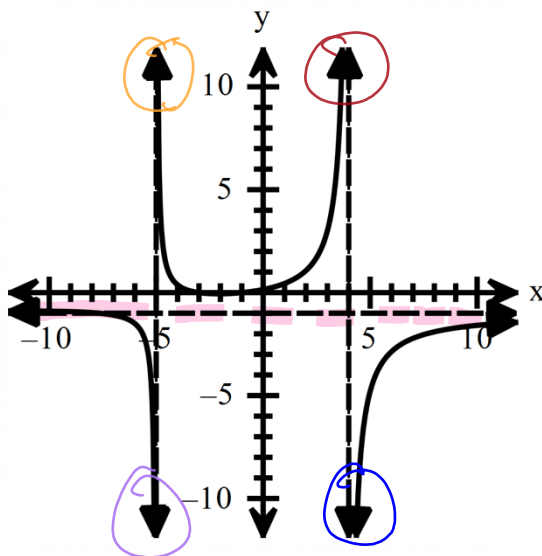
$$\lim_{x \rightarrow -3^-} f(x) = -\infty$$

left side
of asymptote

$$\lim_{x \rightarrow -3^+} f(x) = \infty$$

right side
of asymptote

2.



End Behavior:

$$\lim_{x \rightarrow -\infty} f(x) = -$$

$$\lim_{x \rightarrow \infty} f(x) = +$$

Limits at the asymptotes:

$$\lim_{x \rightarrow 4^-} f(x) = \infty$$

$$\lim_{x \rightarrow 4^+} f(x) = -\infty$$

$$\lim_{x \rightarrow -5^+} f(x) = -\infty$$

$$\lim_{x \rightarrow -5^-} f(x) = \infty$$