

# 6.3

Date: 12/12/23

Objective: I can graph rational functions.

## Writing Equations of Rational Functions

If you are given the graph, you should be able to write a rational equation to match.

Find the asymptotes and intercepts. Then write the vertical asymptotes and  $x$ -intercept(s) as factors. Next be sure your rules for the horizontal asymptotes work. Lastly, make sure your  $y$ -intercept is correct.

### EXAMPLES:

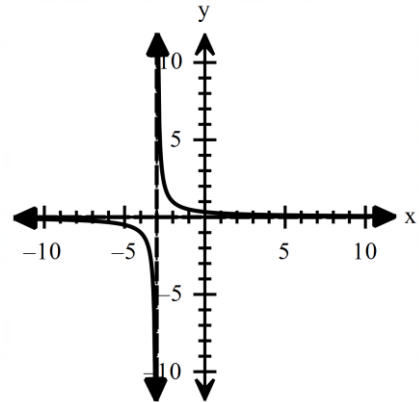
1. Vertical Asymptote:  $x = -3$  — denom

Domain:  $(-\infty, -3) \cup (-3, \infty)$

Horizontal Asymptote:  $y = 0$  —  $n < d$

$x$ -intercept: none — top

Equation:  $f(x) = \frac{1}{x+3}$



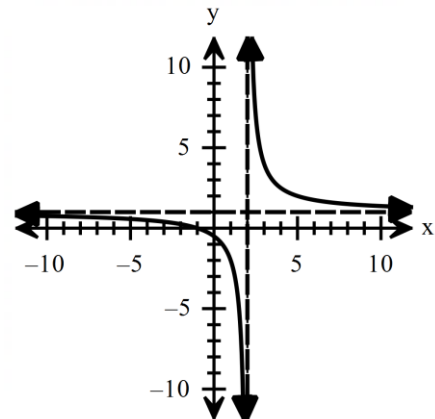
2. Vertical Asymptote:  $x = 2$  bottom

Domain:  $(-\infty, 2) \cup (2, \infty)$

Horizontal Asymptote:  $y = 1$  —  $n = d$

$x$ -intercept:  $(-1, 0)$  top  $x = -1$

Equation:  $f(x) = \frac{x+1}{x-2}$



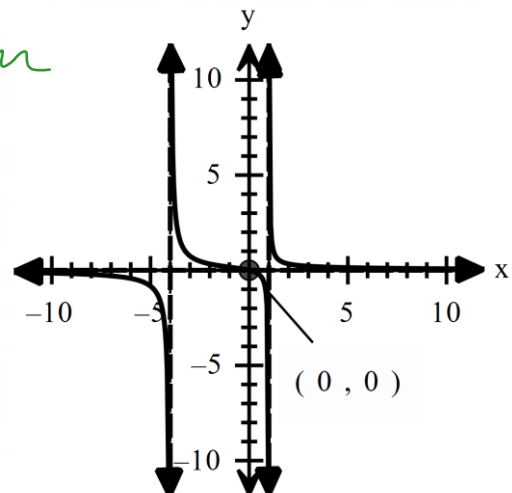
3. Vertical Asymptote:  $x = -4, x = 1$  denom

Domain:  $(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$

Horizontal Asymptote:  $y = 0$  —  $n < d$

$x$ -intercept:  $(0, 0)$  num

Equation:  $f(x) = \frac{x}{(x+4)(x-1)}$



## Graphing Rational Functions

Using the asymptotes and intercepts, you should be able to graph the equation.

Use a sign array to help determine where each section of the rational function is graphed. Plug in different x-values into the equation (depending on asymptotes)....

- If it says positive, that means the y-values are positive in that section. This means in that section the graph is above the x-axis.
- If it says negative, that means the y-values are negative in that section. This means in that section the graph is below the x-axis.

**Example:** Use the following information to graph the rational equations without technology and determine the domain.

4.  $f(x) = \frac{1}{x+8}$

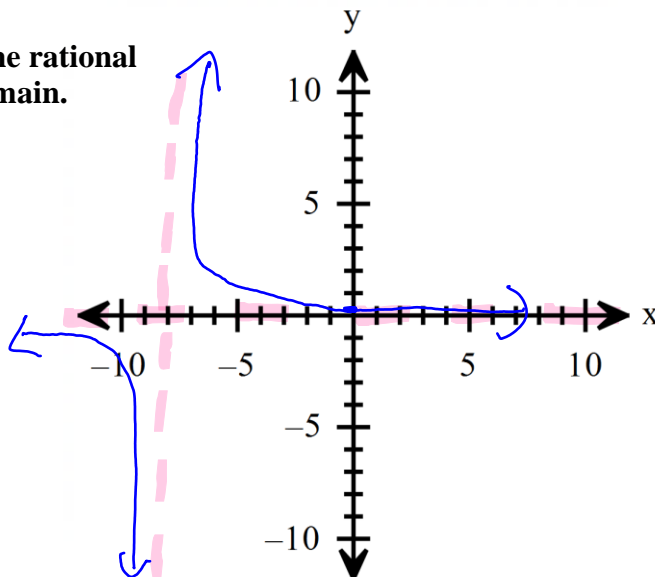
vertical asymptote:  $x = -8$

horizontal asymptote:  $y = 0$

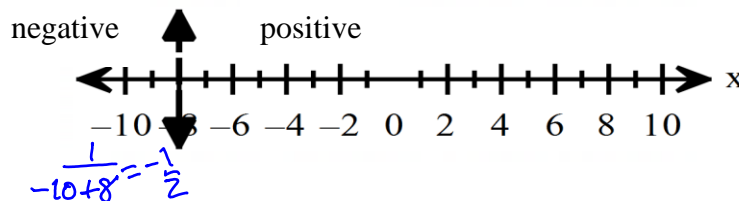
x-intercept: NONE

y-intercept:  $(0, \frac{1}{8})$

Domain:  $(-\infty, -8) \cup (-8, \infty)$



Use the given sign array to help graph the rational function. Describe how to find each piece of the given sign array.



**EXAMPLES:** Find the following information. Then graph the equation.

5.  $f(x) = \frac{1}{x+5}$

Vertical Asymptote:  $x = -5$

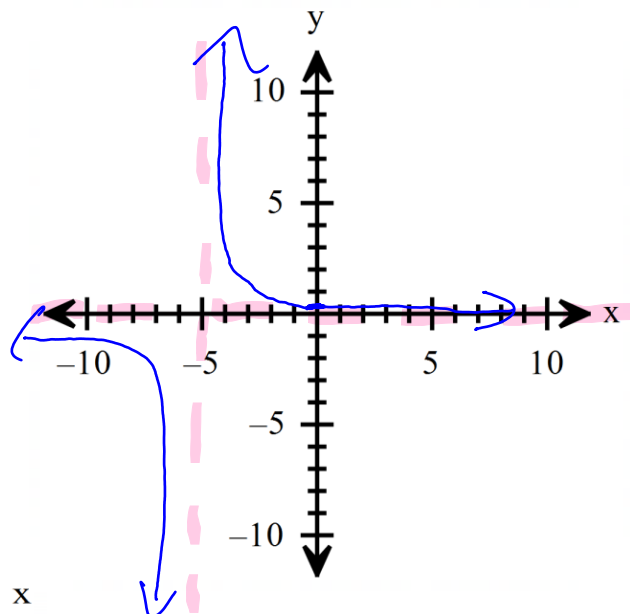
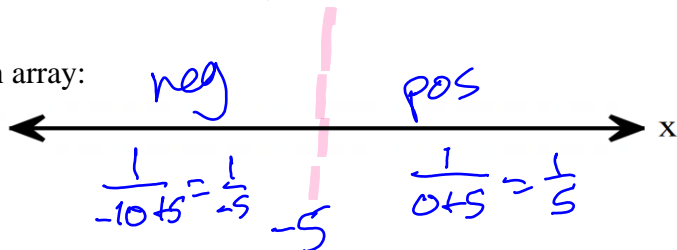
Domain:  $(-\infty, -5) \cup (-5, \infty)$

Horizontal Asymptote:  $y = 0$

x-intercept: none

y-intercept:  $(0, \frac{1}{5})$

Sign array:



$\frac{1}{x+5}$   
 $0 = 1$

6.  $f(x) = \frac{x+4}{x-1}$

Vertical Asymptote:  $x=1$

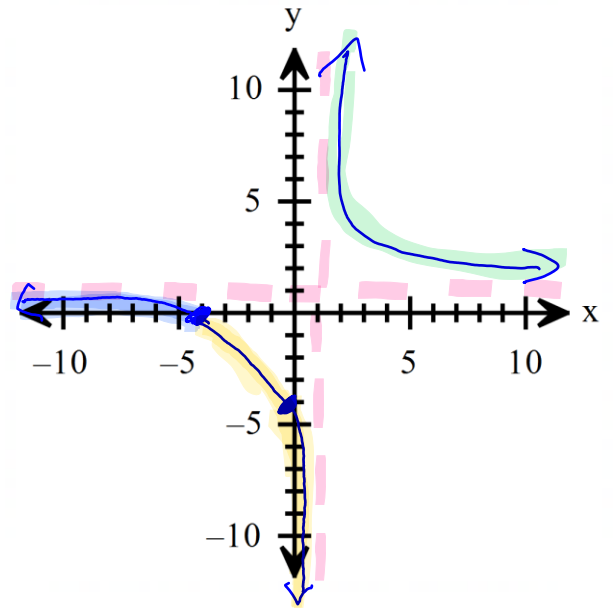
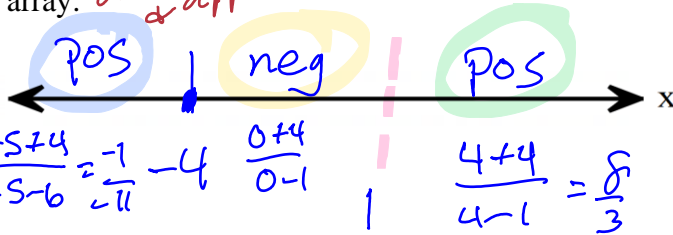
Domain:  $(-\infty, 1) \cup (1, \infty)$

Horizontal Asymptote:  $y=1$

x-intercept:  $(-4, 0)$

y-intercept:  $(0, -4)$   $\frac{0+4}{0-1} = -4$

Sign array: *above x-axis & approach H.A.*



7.  $f(x) = \frac{x-1}{x^2-x-6} = \frac{x-1}{(x-3)(x+2)}$

bottom Vertical Asymptote:  $x=3, x=-2$

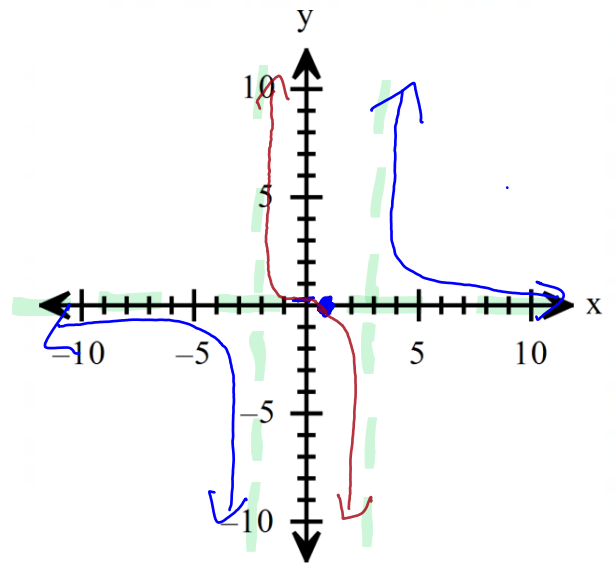
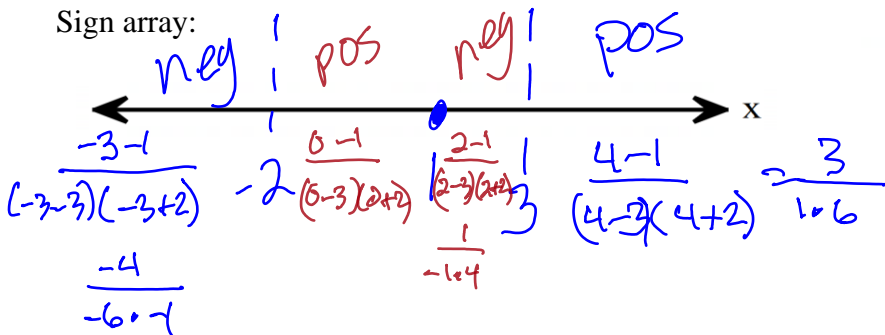
Domain:  $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

Horizontal Asymptote:  $y=0$   $n < d$

top x-intercept:  $(1, 0)$

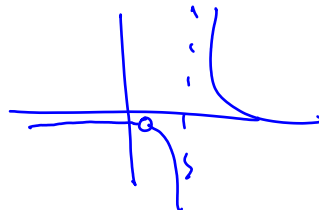
y-intercept:  $(0, \frac{1}{6})$

Sign array:



*☆ in middle you can cross the H.A.*

$\frac{x+1}{(x+1)(x-3)} = \frac{1}{x-3}$   
hole at -1



diagonal asymptote

