

8.3

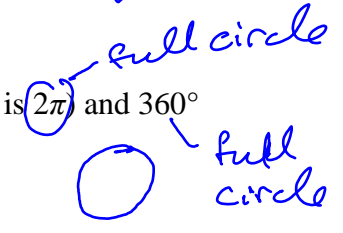
Date: 2/27/24

Objective: I can change radians to degrees and vice versa. I can find arc length and sector area.

1. Converting Between Radians and Degrees:

Since there are 2π radians in a circle (the circumference of the unit circle is 2π) and 360° in a circle,

$$2\pi \text{ radians} = 360^\circ, \text{ or } \pi \text{ radians} = 180^\circ.$$



Degrees → **Radians**: multiply by $\frac{\pi}{180^\circ}$

Radians → **Degrees**: multiply by $\frac{180^\circ}{\pi}$

Examples:

Convert the degree measures to radians. Or convert the radian measures to degrees:

1. $45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4}$



2. $\frac{2\pi}{3} \cdot \frac{180^\circ}{\pi} = 120^\circ$



3. $-\frac{11\pi}{6} \cdot \frac{180^\circ}{\pi} = -330^\circ$

4. $-935^\circ \cdot \frac{\pi}{180^\circ} = -\frac{187\pi}{36}$

5. $3.1 \cdot \frac{180^\circ}{\pi} \approx 177.6^\circ$
no degree sign means it's radians

6. $80^\circ \cdot \frac{\pi}{180^\circ} = \frac{4\pi}{9}$

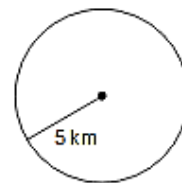
What is the formula for circumference?

$$2\pi r$$

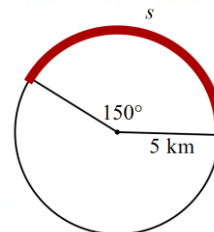
Review example: Find the circumference of the given circle.

$$2\pi 5 = \frac{\text{exact}}{10\pi} \text{ km}$$

$$\text{Round} \approx \underline{31.4 \text{ km}}$$



What if you only want part of the circumference? I only want the bold part of the circumference. How do I find it?



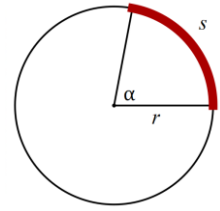
Arc length: part of circumference
fraction of circumference = fraction times circums

Degrees: $2\pi r \left(\frac{\alpha}{360^\circ} \right)$

$\alpha =$ central angle

Radians: $2\pi r \left(\frac{\alpha}{2\pi} \right)$ or $r\alpha$

This formula only works if α is in radians!



Find the arc length. Write the exact answer in terms of π . Then write the rounded answer to the nearest tenth.

<p>1.</p> <p>$2\pi(14)\left(\frac{90^\circ}{360^\circ}\right)$ <u>exact</u> 7π cm <u>round</u> 22.0 cm</p>	<p>2. $r = 17$ cm, $\theta = \frac{7\pi}{6}$ rad; find s</p> <p>$2\pi(17)\left(\frac{7\pi}{6}\right) = \frac{119}{6}\pi$ cm <u>exact</u> $17\left(\frac{7\pi}{6}\right)$ <u>round</u> 62.3 cm</p>
<p>3. $s = 4$ m, $r = 2$ m; find θ</p> <p>$4 = 2\pi(2)\left(\frac{\theta}{2\pi}\right)$ $4 = 2\theta$ $\theta = 2$ rad</p> <p>$4 = 2\pi(2)\left(\frac{\theta}{360^\circ}\right)$ $4 = .0349\theta$ $\frac{4}{.0349} \approx \theta$ $114.6^\circ \approx \theta$</p>	<p>4. $s = 9$ in, $\theta = 6$ rad; find r</p> <p>$9 = 2\pi r \left(\frac{6}{2\pi} \right)$ $9 = 6r$ $\frac{9}{6} = r$ $r = \frac{3}{2} = 1.5$ or $1\frac{1}{2}$ in</p>

What is the area formula of a circle?

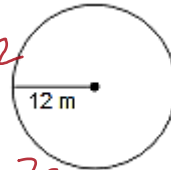
πr^2

$2\pi r$

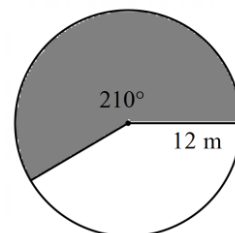
πr^2

Review example: Find the area of the circle.

exact
 $\pi(12)^2 = 144\pi$ m²
round ≈ 452.4 m²



What if you only want part of the area? I only want the shaded part of the area. How do I find it?

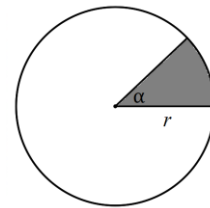


Sector area: part of area or fraction times area

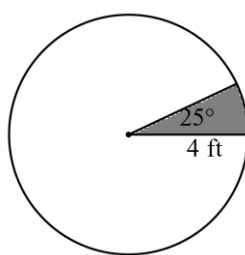
Degrees: $\pi r^2 \left(\frac{\theta}{360^\circ} \right)$

Radians: $\pi r^2 \left(\frac{\theta}{2\pi} \right)$ or $\frac{r^2 \theta}{2}$

This formula only works if α is in radians!

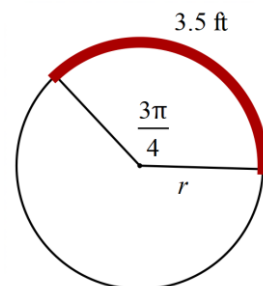


Find the sector area. Write the exact answer in terms of π . Then write the rounded answer to the nearest tenth.

<p>1.</p>  <p>$\pi (4)^2 \left(\frac{25^\circ}{360^\circ} \right)$ <u>exact</u> $\frac{10}{9} \pi \text{ ft}^2$ <u>round</u> $\approx 3.5 \text{ ft}^2$</p>	<p>2. $r = 9 \text{ mi}, \alpha = \frac{\pi}{4}$</p> <p>$\pi 9^2 \left(\frac{\pi}{4} \right)$ <u>exact</u> $\frac{81}{8} \pi \text{ mi}^2$ <u>round</u> 31.8 mi^2</p> <p>$\frac{9^2 \pi}{4}$ $2^2 \div 4 \div 2$</p>
<p>3. $r = 10 \text{ km}, \alpha = 300^\circ$</p>	<p>4. $r = 10 \text{ ft}, \alpha = 60^\circ$</p>

Use the arc length formula and the given information to find the indicated measure. Round answers to the nearest tenth if necessary. Show work using the formula. Remember, leave work and answer in the units it begins with.

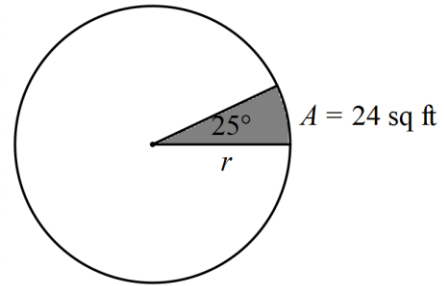
1. $s = 3.5 \text{ ft}, \theta = \frac{3\pi}{4} \text{ rad};$ find r



2. $r = 3 \text{ in}, s = 20.5 \text{ in};$ find θ in (leave answer in degrees)

Use the sector area formula and the given information to find the indicated measure. Round answers to the nearest tenth if necessary. Show work using the formula. Remember, leave work and answer in the units it begins with.

1. $\alpha = 25^\circ$, $A = 24$ sq. ft; find r



2. $r = 3$ ft., $A = 36$ sq. ft.; find α (leave answer in radians)