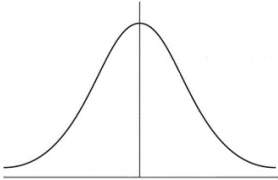# Secondary Math 3
Statistics #2 – Normal Distributions

1/5/24

Normal Distributions
- **Distribution**: how the data varies
- **Normal distribution**: a distribution that is symmetric about the mean of the data. Often represented as a bell curve
- Heights, blood pressure, and many standardized test scores are normal distributions
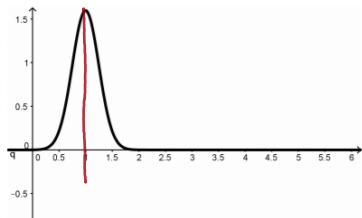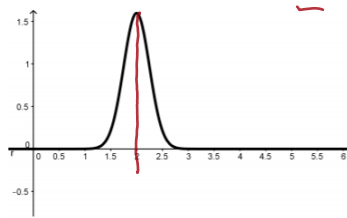
-average

Mean & Standard Deviation
The **mean** is different for each of the curves below. The standard deviation is the same.

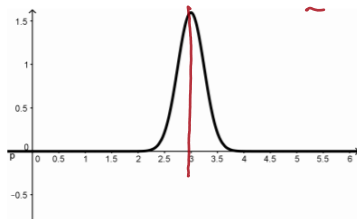Mean = 1, Standard Deviation = 0.25

Mean = 2, Standard Deviation = 0.25
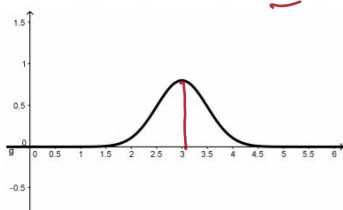
What does the **mean** tell us?

the average
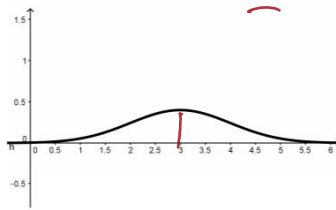
Mean = 3, Standard Deviation = 0.25

The **standard deviation** is different for each of the curves below. The mean is the same.
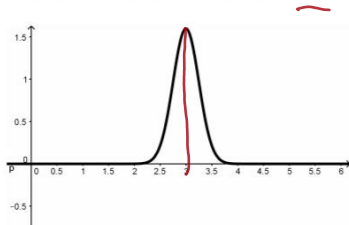
Mean = 3, Standard Deviation = 0.5

Mean = 3, Standard Deviation = 1

What does the **standard deviation** tell us?

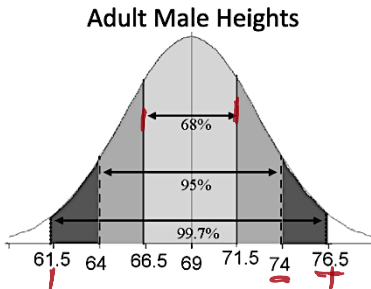*tells how spread out your data is or how it varies*

Mean = 3, Standard Deviation = 0.25

What is standard deviation used for?

To determine how likely an event is, we look at how many standard deviations from the mean it is. For example, suppose the mean height for males is 69 inches with a standard deviation of 2.5.

Z-Scores are measurements of how far from the center (mean) a data value falls.

Adult Male Heights

68%

95%

99.7%

61.5 64 66.5 69 71.5 74 76.5

Ex: A man who stands 71.5 inches tall is 1 standard deviation ABOVE the mean.(z-score = 1)

Ex: A man who stands 64 inches tall is 2 standard deviations BELOW the mean. (z-score = -2)

Z-score represents the exact number of standard deviations a value, x, is from the mean.

observation (value)   mean

$$z = \dfrac{x - \bar{x}}{s}$$

standard deviation

Important Note: We always measure the likelihood of an event based on how many standard deviations the event is from the mean NOT based on the numerical distance between data points.

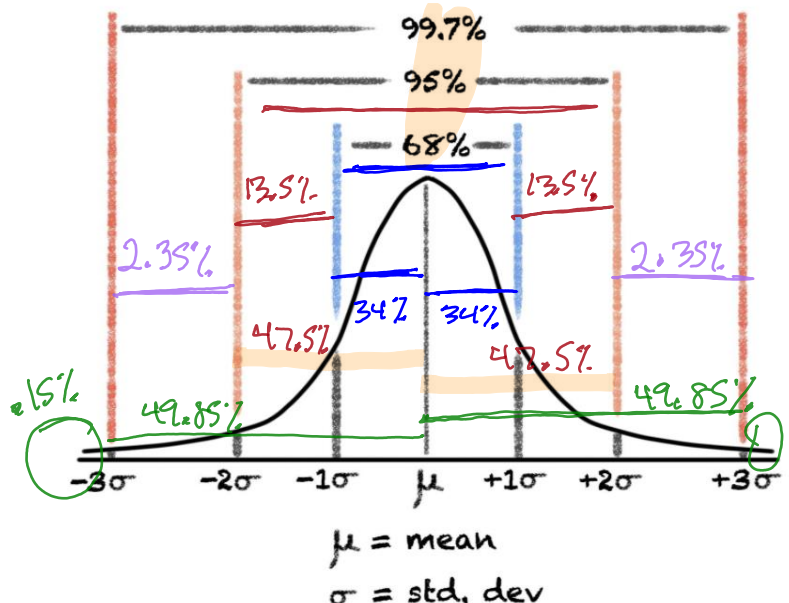Practice: Determine the z-score of each scenario.

Scenario	z-score
- Delivery time for Crumbl Cookies is distributed normally. Suppose the mean delivery time for Crumbl Cookies is 15 minutes with a standard deviation of 2.5 minutes. Your cookies are delivered after 10 minutes. $X = mean = 15 \quad S = \sigma = 2.5 \quad X = observation = 10$	$\dfrac{10 - 15}{2.5} = -2$
- Height of NBA players is distributed normally with a mean of 6'9" and a standard deviation of 2". Your favorite player is 6'7"	$\dfrac{6'7" - 6'9"}{2"} = -1$
- IQ scores are distributed normally. The mean is 100 with a standard deviation of 15. Your IQ is 130	$\dfrac{130 - 100}{15} = 2$

o   Which of the following scenario above is more likely to occur?

68 – 95 – 99.7 Rule

If data is distributed normally, then the following is true:

- 68% of the data points fall within 1 standard deviation of the mean
- 95% of the data points fall within 2 standard deviations of the mean
- 99.7% of the data points fall within 3 standard deviations of the mean

99.7%

95%

68%

13.5%   13.5%

2.35%   2.35%

34%   34%

47.5%   47.5%

.15%

49.85%   49.85%

-3σ   -2σ   -1σ   μ   +1σ   +2σ   +3σ

μ = mean

σ = std. dev

Fill in the normal curve below to show what percent of the data fall into each section.
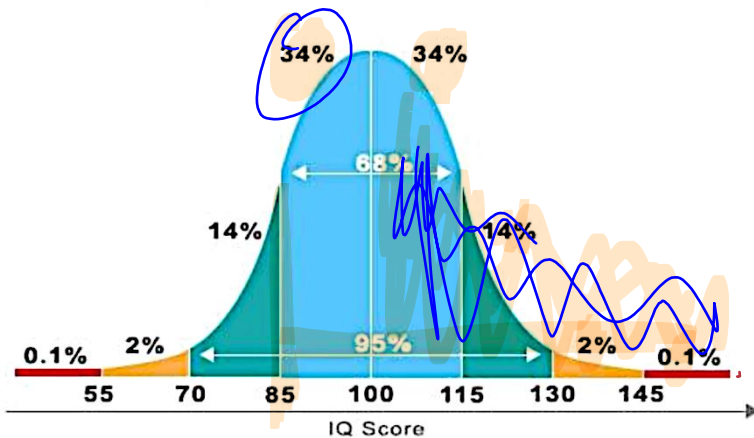

34%

.15%. **2.35%** **13.5%.**

−3σ (−2σ) −1σ μ 1σ 2σ 3σ   X

Example:
The IQ of a population is given. Answer the following questions about the graph.

IQ GRAPH

34% 34%

68%

14% 14%

0.1% 2% 95% 2% 0.1%

55 70 85 100 115 130 145

IQ Score

- What is the mean? What does it represent?
  100   50% have
              100 ± σ
- What is the Standard deviation? What does it represent?   15
                                    IQ
                                    Score
                                    from center
- Is it more likely that a person has a IQ of 85 or an IQ of 115?
        Same amount
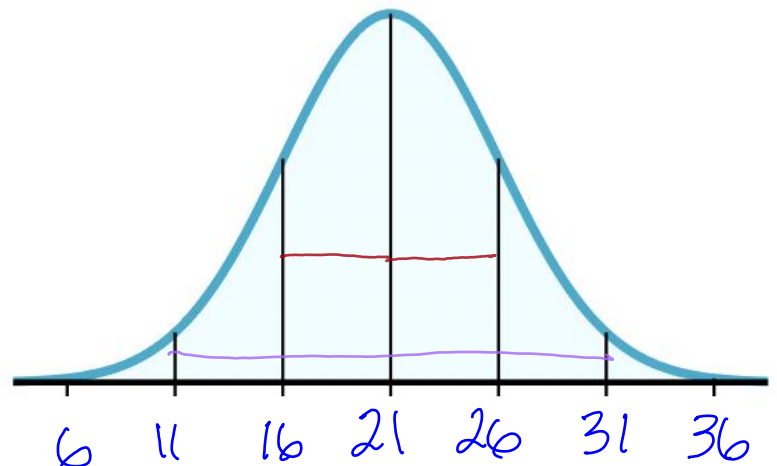- What percent of people have an IQ greater than 85?
        84%.

Example:
Use the following information to identify the parts and percentages of a normal distribution curve.
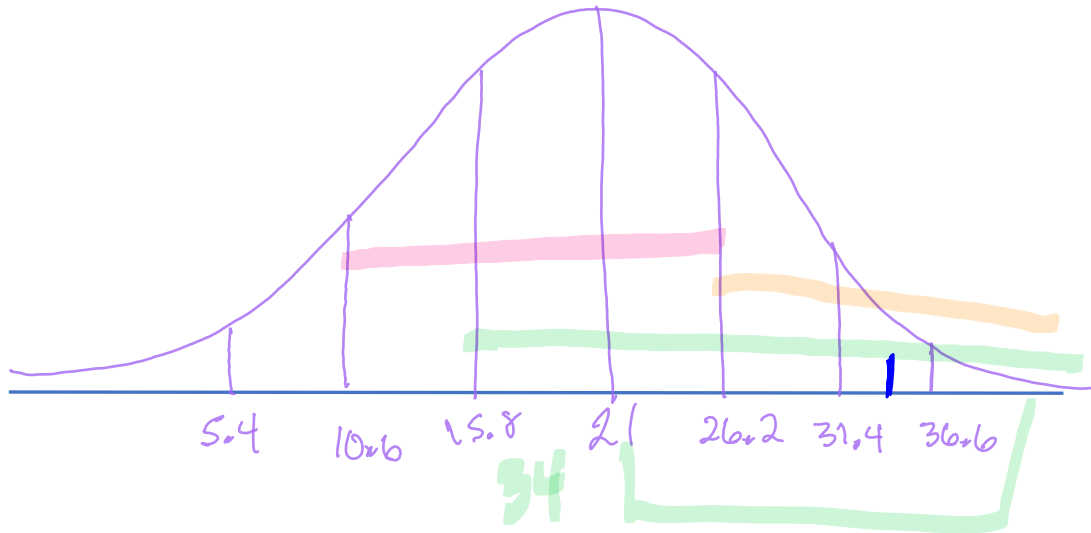
- ACT SCORES
  o Mean: 21
  o Standard deviation: 5

- What percent of people are 1 standard deviation away from the mean?   68%.

- What percent of people are 2 standard deviations away from the mean?   95%.

6 11 16 21 26 31 36

One of the most common examples of a normal distribution is the distribution of scores on standardized tests like the ACT. In 2010, the mean score was 21 and the standard deviation was 5.2 (Source: National Center for Education Statistics).

1. Use this information to sketch a normal distribution curve for this test.

Hand-drawn normal distribution curve with values along the x-axis: 5.4, 10.6, 15.8, 21, 26.2, 31.4, 36.6

2. What percentage of students scored below 21?

   50%

3. What percentage of students scored above 26?

   16%

4. About what percentage of students scored above 16?

   84%

5. About what percentage of students scored between 11 and 26?

   81.5%

6. Your friend, Calvin, would like to go to a very selective college that only admits the top 1% of all student applicants. Calvin has good grades and scored 33 on the test. Do you think that Calvin's ACT score gives him a good chance of being admitted? Explain your answer.

   yes

   or

   no, cuz his score is not the top 1% of scores