SM 3H

DATE: 9/11/23 SECTION: 1.7

OBJECTIVE: I can find all rational and irrational zeros.

Rational Zero Theorem: all factors of constant (p) divide it by all factors of leading coef (q), then put ± on all #'s

$\frac{p}{q}$ gives you all possible rat zeros

Steps:
1. find $\frac{p}{q}$ & put ±
2. use synthetic ÷, long ÷, remainder Thm, or factor
3. repeat until all zeros are found

EXAMPLE:

Use the Rational Zeros Theorem to write a list of all potential rational zeros.

1. $f(x) = x^2 + 7x + 12$ q = 1, p

   ±1 ±2 ±3 ±4 ±6 ±12

   $\frac{p}{1}$ $\frac{p}{3}$

2. $f(x) = 3x^4 - 8x^3 - 37x^2 + 2x + 40$ q = 1, 3, p

   $\pm\frac{1}{1}$ $\pm\frac{2}{1}$ $\pm\frac{4}{1}$ $\pm\frac{5}{1}$ $\pm\frac{8}{1}$ $\pm\frac{10}{1}$ $\pm\frac{20}{1}$ $\pm\frac{40}{1}$

   $\pm\frac{1}{3}$ $\pm\frac{2}{3}$ $\pm\frac{4}{3}$ $\pm\frac{5}{3}$ $\pm\frac{8}{3}$ $\pm\frac{10}{3}$ $\pm\frac{20}{3}$ $\pm\frac{40}{3}$

Use the Rational Zeros Theorem to write a list of all potential rational zeros. Then determine which ones, if any, are zeros. Show your work on a separate piece of paper.

1. $f(x) = x^2 + 7x + 12$ ±1 ±2 ±3 ±4 ±6 ±12

   x = -3, -4

1 | 1 7 12
  1 8
  1 8 20 upper bound

-1 | 1 7 12
  -1 -6
  1 6 6 upper bound

2 | 1 7 12
  2 18
  1 9 30

-2 | 1 7 12
  -2 -10
  1 5 2

$x+5+\frac{2}{x+2}$

3 | 1 7 12
  3 30
  1 10 42

-3 | 1 7 12
  -3 -12
  1 4 0

x + 4 = 0

2. $f(x) = 3x^4 - 8x^3 - 37x^2 + 2x + 40$ $x = 1, -2, 5, -\frac{4}{3}$ $f(x) = (x-1)(x+2)(x-5)(3x+4)$

1) 3 −8 −37 2 40
 3 −5 −42 −40
3 −5 −42 −40 0

$3x^3 - 5x^2 - 42x - 40$

−1) 3 −5 −42 −40
 −3 8 34
3 −8 −34 −6

2) 3 −5 −42 −40
 6 2 −80
3 1 −40 −120

−2) 3 −5 −42 −40
 −6 22 40
3 −11 −20 0

$3x^2 - 11x - 20 = 0$ −60, −15, 4, −11

$(3x+4)(x-5) = 0$

Upper Bounds: if synthetic ÷ ans. are all positive, Then no #'s above the zero will work.

Lower Bounds: if synthetic ÷ ans are alternating, Then no #'s below the zero will work.

~~**Steps:**~~

EXAMPLE: show All work

Prove the real zeros lie in the interval $[-2, 5]$.

1. $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$ $-2x^4 + 7x^3 + 8x^2 - 14x - 8$

5) 2 −7 −8 14 8
 10 15 35 245
2 3 7 49 253

all pos – upper bound

5) −2 7 8 −14 −8
 −10 −15 −35 −245
−2 −3 −7 −49 −253

−2) 2 −7 −8 14 8
 −4 22 −28 28
2 −11 14 −14 32

alt signs – lower bounds

and irrational zeros ($\sqrt{\ }$)

*******Complex zeros **ALWAYS** come in ____pairs____ called conjugates!

If you have $x = 3 + i$, then you must have ___$3-i$___.

$x=\sqrt{5}$ $\quad x=-\sqrt{5}$

$x=1-\sqrt{7}$ $\quad x=1+\sqrt{7}$

EXAMPLE using everything from this lesson to make finding the zeros easier and less work.

Find all of the real zeros of the function, finding exact values whenever possible. Identify each zero as rational or irrational.

1. $f(x) = 2x^3 - 3x^2 - 4x + 6$

$\pm 1, \pm 2, \cancel{\pm 3}, \cancel{\pm 6}, \pm\frac{1}{2}, \pm\frac{3}{2}$

$x = \frac{3}{2}, \sqrt{2}, -\sqrt{2}$

$f(x) = (2x-3)(x+\sqrt{2})(x-\sqrt{2})$

$$\begin{array}{r|rrrr} 1 & 2 & -3 & -4 & 6 \\ & & 2 & -1 & -5 \\ \hline & 2 & -1 & -5 & 1 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 2 & -3 & -4 & 6 \\ & & -2 & 5 & -1 \\ \hline & 2 & -5 & 1 & 5 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -4 & 6 \\ & & 4 & 2 & -4 \\ \hline & 2 & 1 & -2 & 2 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -4 & 6 \\ & & -4 & 14 & -20 \\ \hline & 2 & -7 & 10 & -14 \end{array}$$ lower bound

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -4 & 6 \\ & & 6 & 9 & 15 \\ \hline & 2 & 3 & 5 & 21 \end{array}$$ upper bound

$$\begin{array}{r|rrrr} \frac{3}{2} & 2 & -3 & -4 & 6 \\ & & 3 & 0 & -6 \\ \hline & 2 & 0 & -4 & 0 \end{array}$$

$2x^2 \quad -4 = 0$

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$