SM 3H

DATE: 9/12/23 SECTION: 1.8

OBJECTIVE: I can find ALL zeros, including multiplicities and complex numbers.

Vocabulary

Linear factorization: $(x^2+4) \rightarrow (x-2i)(x+2i)$

Complex numbers:

Standard Form: Example:

Imaginary numbers:

Example: $i =$ ____ $i^2 =$ ____ $i^3 =$ ____ $i^4 =$ ____

Complex conjugate:

If a complex number is a zero, they **always** come in pairs and are conjugates.

EXAMPLES

A) Identify the zeros of the function. B) Find the x-intercepts of its graph. C) Write the polynomial in standard form. Show work!

1. $f(x) = (x+3)(x-4)(x+3i)(x-3i)$

a) $-3, 4, 3i, -3i$

b) $(-3,0)$ $(4,0)$

c) $(x^2-x-12)(x^2+9)$ $f(x) = x^4 - x^3 - 3x^2 - 9x - 108$

$x^4 - x^3 - 12x^2 + 9x^2 - 9x - 108$

A) Write a polynomial function of minimum degree in factored form with real coefficients whose zeros include those listed. B) Find the degree of the polynomial (# of zeros). C) Identify the x-intercepts. Show work!

2. $-3, \boxed{1-4i}$ no comma so all 1 number

a) $f(x) = (x+3)(x-1+4i)(x-1-4i)$

b) 3

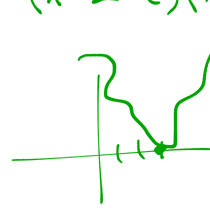
c) $(-3,0)$

3. 3(multiplicity of 2), $2+i$(multiplicity of 1)

a) $f(x) = (x-3)^2(x-2-i)(x-2+i)$

b) 4

c) $(3,0)$



Find all complex zeros of each polynomial. Write the function in factored form. Show work!

4. $f(x) = x^4 - 3x^2 - 4$ $\quad \pm 1 \;\; \pm 2 \;\; \pm 4$

$x = 2, -2, i, -i$

$$\boxed{f(x) = (x-2)(x+2)(x-i)(x+i)}$$

1	1	0	−3	0	−4
		1	1	−2	−2
	1	1	−2	−2	−6

−1	1	0	−3	0	−4
		−1	1	2	−2
	1	−1	−2	2	−6

2	1	0	−3	0	−4
		2	4	2	4
	1	2	1	2	0

−2	1	2	1	2
		−2	0	−2
	1	0	1	0

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm\sqrt{-1}$

$x^4 + 8x^3 + 24x^2 + 32x + 16$ $\quad \pm 1 \pm 2 \pm 4 \pm 8 \pm 16$

$x = -2, -2$

$f(x) = (x+2)^4$

1	1	8	24	32	16
		1	9	33	55
	1	9	33	55	71 — upper

−1	1	8	24	32	16
		−1	−7	−17	−15
	1	7	17	15	1 — upper

−2	1	8	24	32	16
		−2	−12	−24	−16
	1	6	12	8	0

−4	1	6	12	8
		−4	−8	−16
	1	2	4	−8

−8	1	6	12	8
		−8	16	−
	1	−2	28	−

−2	1	6	12	8
		−2	−8	−8
	1	4	4	0

$x^2 + 4x + 4 = 0$

$(x+2)(x+2) = 0$

Using the given zero, find all the remaining zeros of each polynomial. Write the function in factored form. Show work!

5. $2i$ is a zero of $f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$

$x = 2i, -2i, \frac{2}{3}, -\sqrt{2}, \sqrt{2}$

$(x+2i)(x-2i) = x^2 + 4$

$$
\begin{array}{r}
3x^3 - 2x^2 - 6x + 4 \\
x^2 + 0x + 4 \overline{) \, 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16} \\
-\,(3x^5 + 0x^4 + 12x^3) \\
\hline
-2x^4 - 6x^3 - 4x^2 \\
+2x^4 \qquad + 8x^2 \\
\hline
-6x^3 + 4x^2 - 24x \\
+6x^3 \qquad + 24x \\
\hline
4x^2 \quad 0 + 16 \\
-4x^2 \quad -16 \\
\hline
0
\end{array}
$$

$\frac{2}{3}$	3	−2	−6	4
		2	0	−4
	3	0	−6	0

$3x^2 - 6 = 0$

$3x^2 = 6$

$x^2 = 2$

$$\boxed{f(x) = (x-2i)(x+2i)(3x-2)(x-\sqrt{2})(x+\sqrt{2})}$$