SM 3H **Date:** 9/28/23 **Section:** 2.2

Objective: I can write a function using another function.

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example: Find the distance between $(-1,5)$ and $(7,-1)$.

Graphing Circles Review $x^2 + y^2 = r^2$ $(x-h)^2 + (y-k)^2 = r^2$ $(0,0)$

Rectangle
$P = 2l + 2w$
$A = lw$ or bh
$V = lwh$

square
$P = 4s$
$A = s^2$

triangle
$P = s_1 + s_2 + s_3$
$A = \frac{bh}{2}$

Real-world problems often result in mathematical models that involve functions. These functions need to be constructed or built based on the information given. In constructing functions, we must translate the verbal description into the language of math. We do this by assigning symbols to represent the independent and dependent variables and then by finding the function or rule that relates these variables.

Examples:

Let $P = (x, y)$ be a point on the graph of $y = x^2 - 3$.

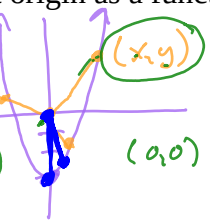
a) Express the distance d from point P to the point origin as a function of x.

$\sqrt{(x-0)^2 + (y-0)^2}$ $(x^2-3)^2$

$\sqrt{x^2 + (x^2-3-0)^2}$

$\sqrt{x^2 + x^4 - 6x^2 + 9}$

$\sqrt{x^4 - 5x^2 + 9} = d$



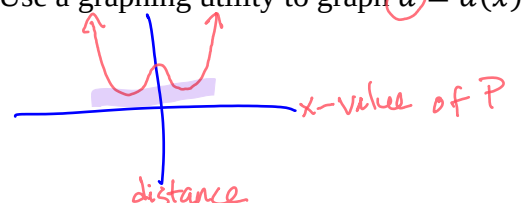
b) What is d if $x = 0$?

$d = \sqrt{0^4 - 5 \cdot 0^2 + 9} = 3$

c) If $x = 1$?

$\sqrt{1^4 - 5 \cdot 1^2 + 9} = \sqrt{5} \approx 2.24$ = exact = round

d) Use a graphing utility to graph $d = d(x)$. Sketch the graph.



e) For what values of x is d smallest?

$x = 1.58, -1.58$

A rectangle has one corner in quadrant I on the graph of $y = 9 - x^2$, another at the origin, a third on the positive x-axis, and a fourth on the positive y-axis.

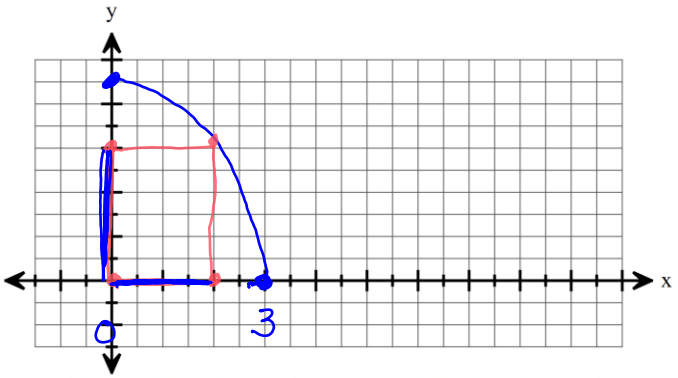
$0 = 9 - x^2 \quad x = \pm 3$

a) Express the area A of the rectangle as a function of x.

$A = bh = xy$

$A = x(9 - x^2)$

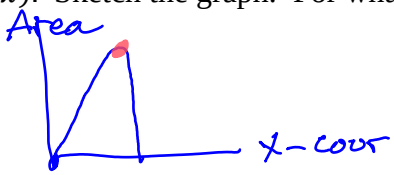
$A = 9x - x^3$



b) What is the domain of A?

$(0, 3)$

c) Graph $A = A(x)$. Sketch the graph. For what value of x is the area largest?



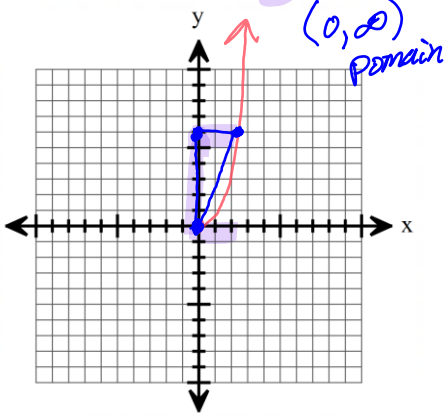
$x \approx 1.73$

$A \approx 10.39$

A right triangle has one vertex on the graph $y = x^9$, $x > 0$, at (x, y), another at the origin, and the third on the positive y-axis at $(0, y)$. Express the area, A, of the triangle as a function of x.

$A = \frac{1}{2}bh = \frac{1}{2}xy$

$A = \frac{1}{2}x^{10}$

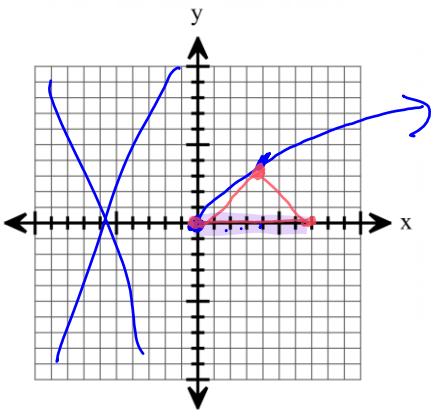


An isosceles triangle has its base along the positive x-axis with one vertex at the origin, another on the x-axis, and a third on the graph of $y = 4\sqrt{x}$. Express the area, A, of the triangle as a function of the *length of the base*.

$A = \frac{1}{2}bh = \frac{1}{2}xy$

$A = \frac{1}{2} \cdot 4\sqrt{x}$

$A = 2x\sqrt{x}$

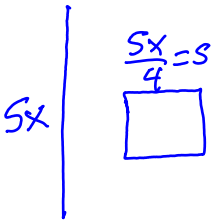


A wire of length $5x$ is bent into the shape of a square.

a) Express the perimeter of the square as a function of x.

$P = 4s = 4\left(\frac{5x}{4}\right)$

$P = 5x$

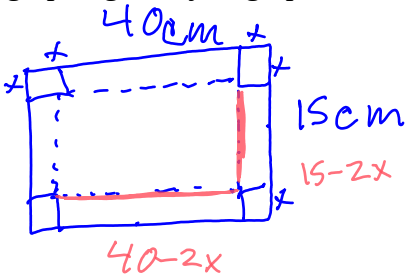


b) Express the area of the square as a function of x.

$A = s^2 = \left(\frac{5x}{4}\right)^2$

$A = \frac{25x^2}{16}$

Squares of width x are removed from a 15 cm by 40 cm piece of cardboard, and the resulting edges are folded up to form a box with no top. Determine all values of x so that the volume of the resulting box is at most 190 cm^3. Round to the nearest hundredths. Write your answer in interval notation. Explain your answer in words. Remember that if it doesn't factor, use a graphing utility to graph.



Domain = $(0, 7.5)$ height = x

$V = lwh$

$(15-2x)(40-2x)x \leq 190$

$(15-2x)(40-2x)(x) - 190 \leq 0$

$(0, 3.4\text{ cm}] \cup [6.98\text{ cm}, 7.5\text{ cm})$