



Date: 10/5/23

Section: 2.5

Objective: I can graph functions using transformations.

What are the 4 types of transformations?

1. Rotation - don't do with functions
2. reflection - flip
3. dilation - stretch/shrink
4. translations - move \leftrightarrow \updownarrow (shift)

Combining transformations - Transformations may be performed in succession - one after another. Pay attention to the order of the transformations....it makes a difference.

When graphing a transformed graph based on the equation of the function, apply transformations in the following order:

1. reflection
2. dilation
3. translations (+/-)

General transformation equation in function notation:

$$f(x) = a f(b(x-h)) + k$$

a & k go with y *b & h go with x but opp*

a = vert. dilation
if a is neg = reflection over x-axis
b = horizontal dilation (opp)
if b is neg = reflect over y-axis
h = translate right/left
k = translate up or down

Examples: List the transformations in the appropriate order:

Parent graph: $y = \sqrt{x}$

a) $y = -\frac{1}{2}\sqrt{x+3}$

a *x-h*

- reflect over x-axis
- vert stretch of $\frac{1}{2}$
- translate left 3

b) $y = \sqrt{-2x+9} = \sqrt{-2(x-4.5)}$

a *b* *x-h* *k*

- reflect over y-axis
- hor stretch by $\frac{1}{2}$
- translate right 4.5

Parent graph: $f(x) = |x|$

a) $f(x) = -\left|\frac{1}{3}(x+6)\right|$

a *b* *h*

- vert reflection
- hor stretch of 3
- translate left 6

b) $f(x) = -|x+5|-3$

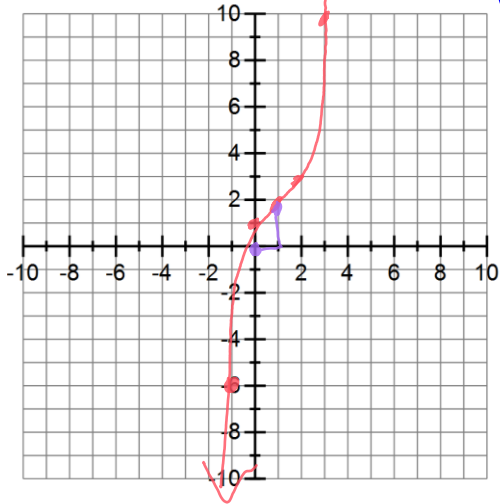
a *h* *k*

- vert reflect
- translate left 5, down 3

Examples: Name the parent graph. Describe how the graph is transformed. Graph the equation using 5 key points:

cubic

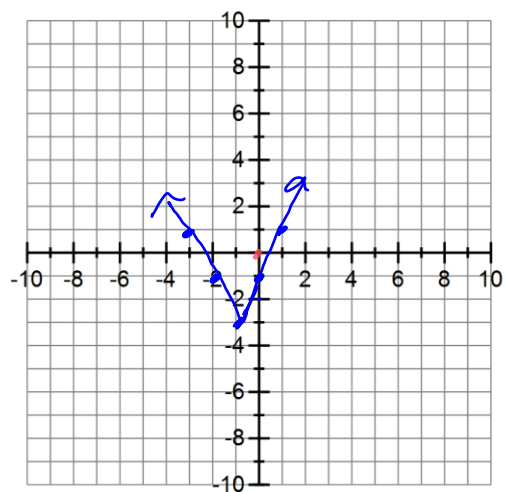
a) $f(x) = (x-1)^3 + 2$



-2	-8
-1	-1
0	0
1	1
2	8

$x+1$	$y+2$
-1	-6
0	1
1	2
2	3
3	10

b) $g(x) = 2|x+1| - 3$

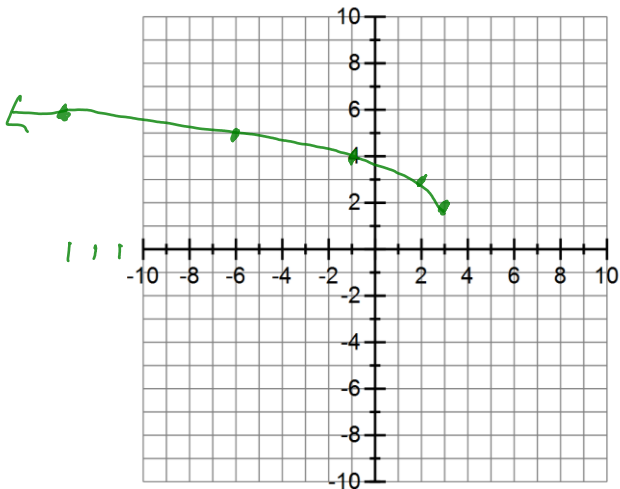


-2	2
-1	1
0	0
1	1
2	2

$x-1$	$2y-3$
-3	1
-2	1
-1	1
0	1
1	1

square root

c) $f(x) = \sqrt{-(x-3)} + 2$

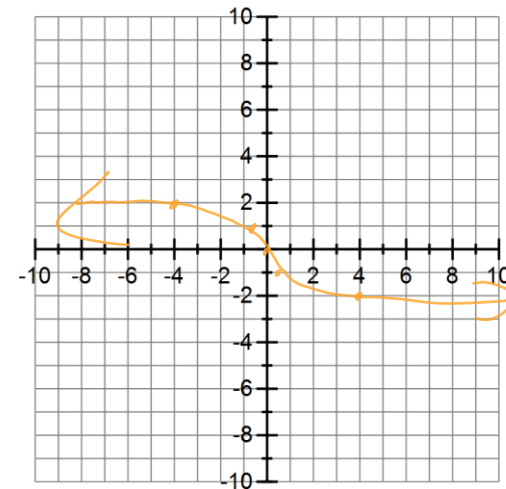


0	0
1	1
4	2
9	3
16	4

$-x+3$	$y+2$
3	2
2	3
-1	4
-6	5
-13	6

cube root

d) $g(x) = \sqrt[3]{2x}$



-8	-2
-1	-1
0	0
1	1
8	2

$\frac{1}{2}x$	-y
-4	2
-1	1
0	0
1	1
2	2

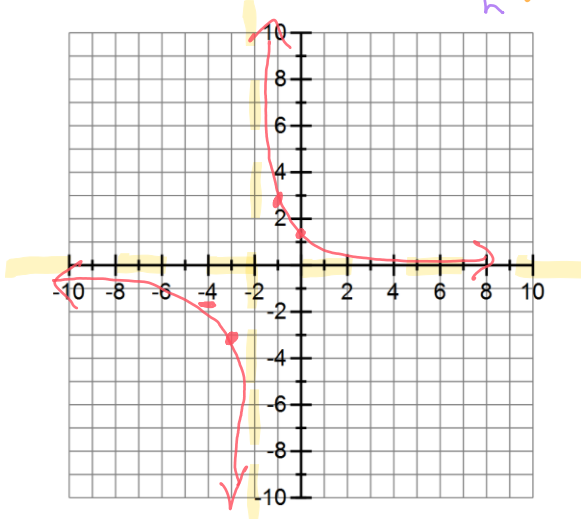
rational

$$e) h(x) = \frac{3}{(x+2)} = 3 \left(\frac{1}{x+2} \right)$$

a h

vert stretch of 3
translate left 2

$\frac{3}{x+2}$



		$x-2$	$3y$
-2	$-\frac{1}{2}$	-4	$-1\frac{1}{2}$
-1	-1	-3	-3
0	undef	-2	undef
1	1	-1	3
2	$\frac{1}{2}$	0	$1\frac{1}{2}$

Summary of Graphing Transformations:

To Graph:	Draw the Graph of $y = f(x)$ and:	Functional Change to $y = f(x)$:
Reflection About the x-axis $y = -f(x)$	Reflect the graph of f about the x -axis.	Multiply $f(x)$ by -1 .
Reflection About the y-axis $y = f(-x)$	Reflect the graph of f about the y -axis.	Replace x by $-x$.
Vertical Stretches & Compressions $y = af(x), a > 0$	Multiply each y -coordinate of $y = f(x)$ by a . This stretches the graph of f vertically if $a > 1$. This compresses the graph of f vertically if $0 < a < 1$.	Multiply $f(x)$ by a .
Horizontal Stretches & Compressions $y = f(bx), b > 0$	Divide each x -coordinate of $y = f(x)$ by b . This stretches the graph of f horizontally if $0 < b < 1$. This compresses the graph of f horizontally if $b > 1$.	Replace x by bx .
Vertical Shifts $y = f(x) + k, k > 0$ $y = f(x) - k, k > 0$	Raise the graph of f by k units. Lower the graph of f by k units.	Add k to $f(x)$ Subtract k from $f(x)$
Horizontal Shifts $y = f(x - h), h > 0$ $y = f(x + h), h > 0$	Shift the graph of f to the right by h units. Shift the graph of f to the left by h units.	Replace x by $x - h$. Replace x by $x + h$.

EXAMPLE: Describe a basic graph and a sequence of transformations that can be used to produce a graph of the given function.

a) $y = -2\sqrt{x+3}$

EXAMPLE: A new graph is obtained from the series of transformations on the given graph; write the equation for the new graph.

- a) Starting with $y = \sqrt{x}$, reflect across the x-axis, vertical stretch by factor of 2, and shift left 3.

$$f(x) = -2\sqrt{x+3}$$

- b) Starting with $y = x^2$; a vertical stretch by a factor of 4, then a shift right 6 units.

$$f(x) = 4(x-6)^2$$