SM 3H

Date: 10/9/23 **Section:** 2.6

Objective: I can find equations for piecewise functions.
I can analyze a piecewise function.

Graphing Piecewise-Defined Functions

Sometimes a function is defined differently on different parts of its domain. When functions are defined by more than one equation, they are called ***piecewise-defined functions***.

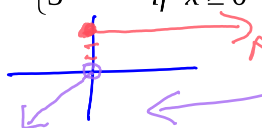
Remember how to graph: $a(x-h)^2+k$

Line $y = mx + b$ (slope, y-int) **Quadratic** **Square root** $a\sqrt{x-h} + k$

Here is how to write a piecewise function.

$$f(x) = \begin{cases} x & \text{if } x < 0 \\ 3 & \text{if } x \geq 0 \end{cases}$$

This means: This 1st eq. is y=x until you hit 0. This is a ○ not a ● because it is not =.
The second eq. is y=3 at 0 then on forever



When you graph a piecewise function, think of the points where the graph changes to the second function as a fence. If you have __<__ or __>__ the function does NOT live on the fence. If you have __≤__ or __≥__ then the function lives on the fence.

EXAMPLE: A) find domain (try to do it without graphing), B) Find the intercept(s), C) Graph the function, D) Find the range.

$$f(x) = \begin{cases} x+1 & \text{if } x \leq -1 \\ x^2+1 & \text{if } x > -1 \end{cases}$$

linear

quadratic

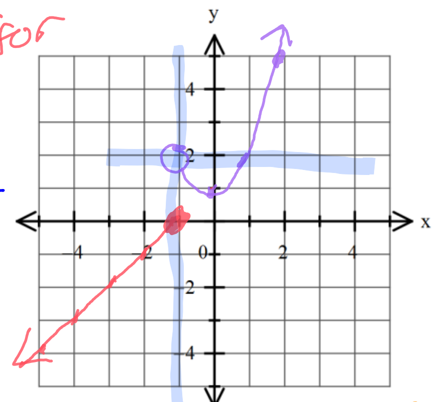
both have (−∞,∞) for domain

since # is same & there is = the domain is all #'s

Domain: (−∞, ∞)

Intercept(s): x-int (−1,0) y-int (0,1)

Range: (−∞,∞)



draw 1st eq & erase parts not needed
draw 2nd eq & erase parts not needed
put circle since no =

even though 0 there is a dot somewhere else on that line so it is continuous

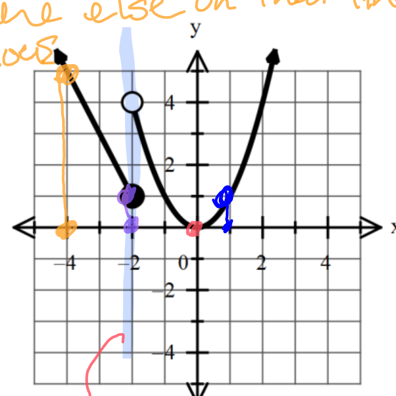
Sometimes we want to find the value at a given coordinate.
You can do this using a graph.

EX. A) $f(-4) =$ 5 when x=−4 y=?

B) $f(-2) =$ 1

C) $f(0) =$ 0

D) $f(1) =$ 1



make sure one is a dot and one is circle so it will pass vertical line test & be a function

We can also find the value by substituting the given coordinate into the equation.

EX. $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ 2x^2 - 4 & \text{if } x > 1 \end{cases}$

A) $f(-10)$

-10 is in domain of 2x, sub in 10

$2 \cdot 10 = \boxed{20}$

B) $f(-2)$

-2 is in domain of 2x

$2(-2) = \boxed{-4}$

C) $f(1)$

1 is in domain of y = 1 so it is a dot at (1, 1)

$\boxed{1}$

D) $f(5)$

5 is in domain of $2x^2-4$ so plug in 5 for x

$2(5)^2 - 4 = \boxed{46}$

Examples: For the following functions:

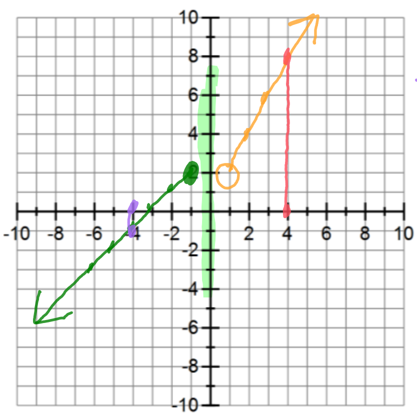
a) Graph the function. — graph each function separately then erase part not in domain & watch O or ●

b) Find the domain and range of the function.

c) Locate any intercepts.

d) State whether the function is continuous on its domain.

1) $f(x) = \begin{cases} x+3 & \text{if } x \le -1 \\ 2x & \text{if } x > 1 \end{cases}$



x-int (-3, 0)

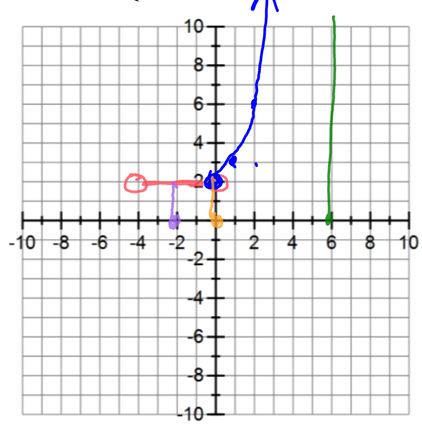
no y-int

Find: $f(-4)$ $f(0)$ $f(4)$

-1   DNE $\boxed{8}$

2) $f(x) = \begin{cases} 2 & \text{if } -4 < x < 0 \\ x^2 + 2 & \text{if } x \ge 0 \end{cases}$

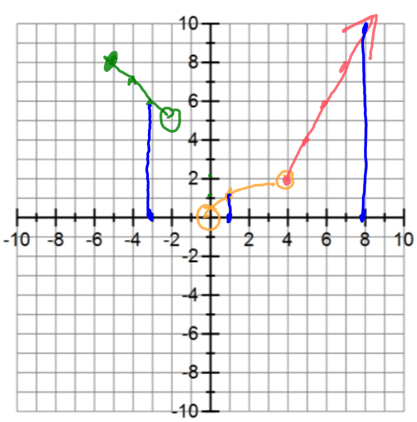
since red is O & blue is ● it is a function but if both were ● or (≤, ≥) then not a function so much have 1 of each on same x



Find: $f(-2) = 2$ $f(0) = 2$ $f(6) = 38$

not on graph so use eq. $6^2 + 2$

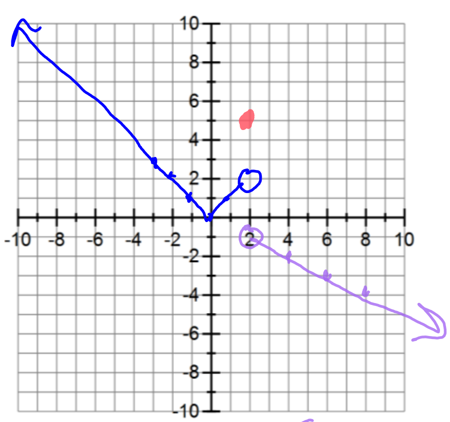
3) $f(x) = \begin{cases} 3-x & \text{if } -5 \le x < -2 \\ \sqrt{x} & \text{if } 0 < x < 4 \\ 2x-6 & \text{if } x \ge 4 \end{cases}$



Find: $f(-3) = 6$ $f(1) = 1$ $f(8) = 10$

$3 + 3 = 6$ $\sqrt{1} = 1$ $2(8) - 6 = 10$

4) $f(x) = \begin{cases} |x| & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ -\frac{1}{2}x & \text{if } x > 2 \end{cases}$

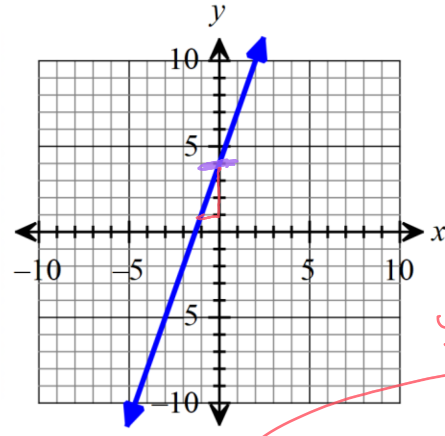


Find: $f(-5) = 5$ $f(2) = 5$ $f(6) = -3$

$|-5|$ $x = 6$, $-\frac{1}{2}(6)$

Review:

Find the equation of the following graphs.

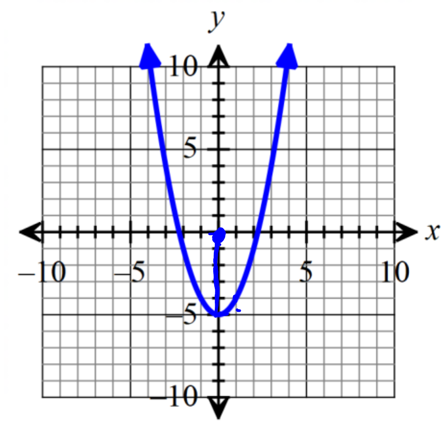


$b = 4$

$m = \frac{-3}{1}$

$y = mx + b$

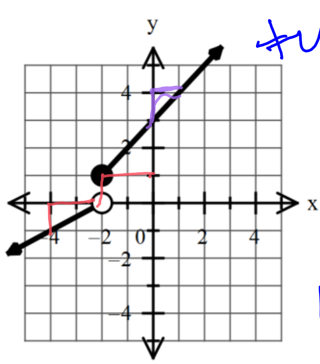
$y = -3x + 4$



down 5

$x^2 - 5 = y$

Write a definition (equation) for each piecewise function.

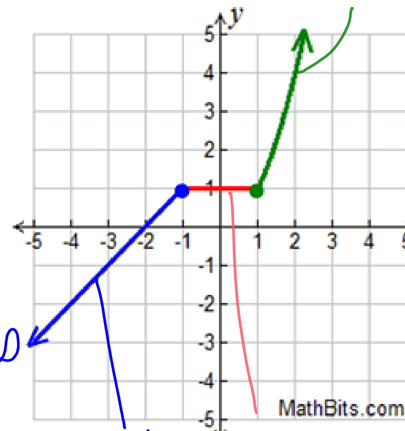


two pieces so 2 eq.

find the eq. & domain

multiple answers can occur.

These can be lines or half an absolute value



parabola = parent x^2

$$f(x) = \begin{cases} x+2 & x \le -1 \\ 1 & -1 < x < 1 \\ x^2 & x \ge 1 \end{cases}$$

$m = 1$
$b = 2$

$y = 1$
since you see green & blue red is 0 on ≠

$\frac{1}{2} = m$

$b = 1$

1st eq $y = \frac{1}{2}x + 1$

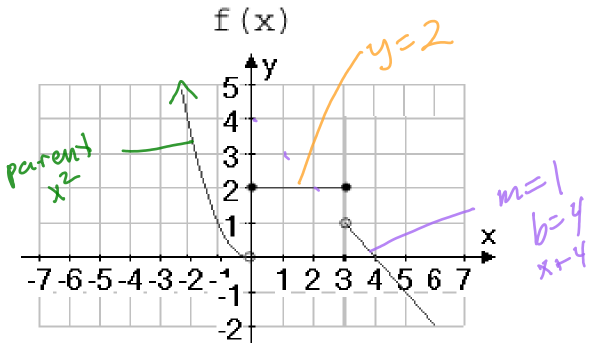
$m = 1$

$b = 3$

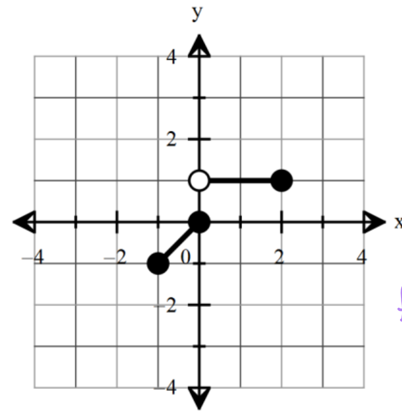
2nd eq $y = x + 3$

	eq	domain
So $f(x) =$	$\frac{1}{2}x + 1$	$x < -2$
	$x + 3$	$x \ge -2$

or $f(x) = \begin{cases} \frac{1}{2}\lvert x+2 \rvert & x < -2 \\ \lvert x+2 \rvert + 1 & x \ge -2 \end{cases}$



$$f(x) = \begin{cases} x^2 & x < 0 \\ 2 & 0 \le x \le 3 \\ x+4 & x > 3 \end{cases}$$



$$f(x) = \begin{cases} x & -1 \le x \le 0 \\ 1 & 0 < x \le 2 \end{cases}$$

OR

$$f(x) = \begin{cases} -|x| & -1 \le x \le 0 \\ 1 & 0 < x \le 2 \end{cases}$$

Here is a real -life example for using a piecewise function. Draw a graph.
A doctor's fee is based on the length of time.

A) up to 6 minutes it costs \$50 — $y = 50 \quad x < 6$

B) over 6 to 15 minutes costs \$80 — $y = 80 \quad 6 \le x \le 15$

C) over 15 minutes \$80 plus \$5 per minute above 15 minutes

$y = 80 + 5x \quad x > 15$

Write the equation.

$$f(x) = \begin{cases} 50 & x < 6 \\ 80 & 6 \le x \le 15 \\ 5x + 80 & x > 15 \end{cases}$$

How much it would cost at 12 minutes?

\$80

How much would it cost for 20 minutes?

5 minutes above 15 so use 5 not 20

$5(5) + 80 = \$105$