SM 3H

DATE: 11/1/23 SECTION: 3.3

OBJECTIVE: I can analyze a rational function.

Vertical Asymptotes: line that goes up & down that function approaches but never touches

How many can you have? infinite

How do you find V. A.? Set denom = 0 & solve

Vertical asymptotes are the restrictions.

EX.

$\frac{5x+1}{2x^2-x-3}$

$2x^2-x-3 \neq 0$

$(2x-3)(x+1) \neq 0$

$x \neq \frac{3}{2}, -1$ = restrictions to find domain

$x = \frac{3}{2}$
$x = -1$
V.A.

Horizontal Asymptotes: line that goes right & left that function approaches but never touches

Oblique Asymptotes: diagonal line that function approaches but never touches

How many can you have? only one

How do you find H. A. or O.A.?

1. deg num < deg denom $y = 0$
2. deg num = deg denom $y =$ leading coef (top & bottom)
3. deg num > deg denom oblique long division to find eq.

EX.

a) $\frac{5x+6}{2x^2-1}$ n < d

$y = 0$

b) $\frac{x^2+3x-4}{x+1}$ n > d

$$\begin{array}{r} x+2 \\ x+1 \overline{) x^2+3x-4} \\ -x^2-x \\ \hline 2x-4 \end{array}$$

$y = x+2$

c) $\frac{9x^2-4}{2x^2-x-3}$ n = d

$y = \frac{9}{2}$

x-intercept(s): where function touches x-axis

How many can you have? infinite

How do you find the x-intercept(s)? set eq = 0 & solve for x

☆ short cut = only use num

EX.

$$\left(\frac{3x+1}{2x^2-1} = 0\right)(2x^2-1)$$

$$\left(-\tfrac{1}{3}, 0\right)$$

$$3x+1=0$$

y-intercept(s): where function touches y-axis

How many can you have? 1

How do you find the y-intercept(s)? set x=0 & solve for y

☆ short cut = use constants (top & bottom)

EX.

$$\frac{x^2+3x+2}{4x^2-1}$$

$$\frac{0^2+3(0)+2}{4(0)^2-1}$$

$$(0, -2)$$

Hole(s): points in graph that don't exist

How many can you have? infinite

How do you find the hole(s)? if a factor crosses, this is x-coor of of hole

to find y-coor, plug x-coor into <u>simplified</u> function & evaluate

EX.

$$\frac{5x^2-5}{x^2+4x+3} = \frac{5(x+1)(x-1)}{(x+3)(x+1)} = \frac{5(x-1)}{x+3}$$

$$(-1, -5)$$

$$y = \frac{5(-1-1)}{-1+3}$$

Sign Arrays: way to know if y-values are pos or neg so you know if graph is above or below x-axis

How do you make a sign array? graph V.A, x-int, holes on number line. Then test each section

EX.

$\frac{x+1}{x^2-4} = \frac{x+1}{(x+2)(x-2)}$

V.A: $x=2$, $x=-2$

x-int: $(-1,0)$

Number line: $-3\ -2\ -1\ 0\ 1\ 2\ 3$ — neg | pos | neg | pos

$\frac{-3+1}{(-3+2)(-3-2)}$

$\frac{-2}{-1\cdot -5} = \text{neg}$

EX. Find the asymptotes, intercepts, and make a sign array for the following function.

a) $f(x) = \frac{3x+4}{x^2-x-12} = \frac{3x+4}{(x-4)(x+3)}$

V.A.: $x=4$, $x=-3$

x-int: $\left(-\frac{4}{3}, 0\right)$

H.A: $y=0$

y-int: $\left(0, -\frac{1}{3}\right)$

Number line: $-4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4$ — neg | pos | neg | pos

$\frac{3(-4)+4}{(-4-4)(-4+3)}$ $\frac{3(-2)+4}{(-2-4)(-2+3)}$ $\frac{3(5)+4}{(5-4)(5+3)}$

b) $f(x) = \frac{x^2+3x-4}{x-6} = \frac{(x+4)(x-1)}{x-6}$

$$\begin{array}{r} x+9 \\ x-6\overline{)x^2+3x-4} \\ \underline{-x^2+6x} \\ 9x-4 \end{array}$$

V.A.: $x=6$

O.A.: $y=x+9$

x-int: $(-4,0)$, $(1,0)$

y-int: $\left(0, \frac{2}{3}\right)$

Number line: $-4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5\ 6$ — neg | pos | neg | pos

$\frac{(7+4)(7-1)}{7-6}$

c) $f(x) = \frac{2x-6}{2x^2-5x-3} = \frac{2(x-3)}{(2x+1)(x-3)} = \frac{2}{2x+1}$

Number line: $-3\ -2\ -1\ 0\ 1\ 2\ 3\ 4$ — neg | pos | pos

V.A.: $x=-\frac{1}{2}$

holes: $\left(3, \frac{2}{7}\right)$

X-int: none

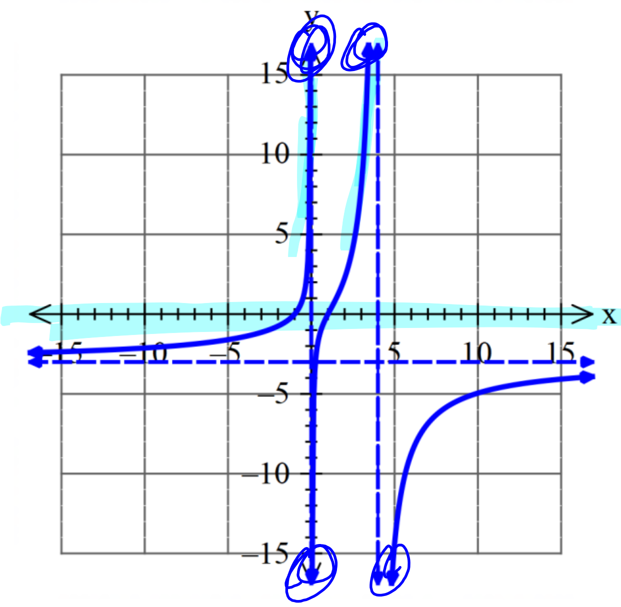
$\boxed{\frac{2}{2\cdot 3+1}}$

y-int: $(0,2)$

H.A: $y=0$

EX. Identify key features of a rational function.

a)



Domain: $(-\infty, 0)\cup(0, 4)\cup(4, \infty)$

Range: $(-\infty, \infty)$

x-intercept(s): $(-1, 0)$ $(1, 0)$

y-intercept: none

Increasing: $(-\infty, 0)\cup(0, 4)\cup(4, \infty)$

Decreasing: N/A

Constant: N/A

Positive: $(-1, 0)\cup(1, 4)$

Negative: $(-\infty, -1)\cup(0, 1)\cup(4, \infty)$

Maximums / minimums:

Symmetry:

End Behavior/Limits:

$\lim_{x\to-\infty} f(x) = -3$ $\lim_{x\to\infty} f(x) = -3$

$\lim_{x\to 0^-} f(x) = \infty$ $\lim_{x\to 0^+} f(x) = -\infty$

$\lim_{x\to 4^-} f(x) = \infty$ $\lim_{x\to 4^+} f(x) = -\infty$

Vertical Asymptote(s):

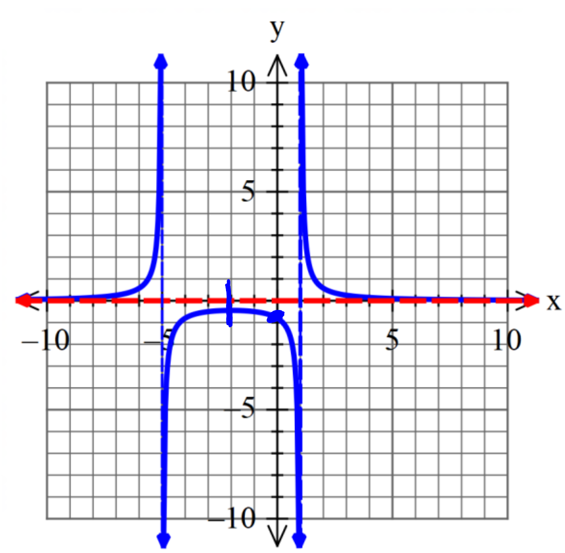
$x = 0$

$x = 4$

Horizontal Asymptote:

$y = -3$

b)



Domain: $(-\infty, -5)\cup(-5, 1)\cup(1, \infty)$

Range: $(-\infty, -.4)\cup(0, \infty)$

x-intercept(s): none

y-intercept: $(0, -.8)$

Increasing: $(-\infty, -5)\cup(-5, -2)$

Decreasing: $(-2, 1)\cup(1, \infty)$

Constant: N/A

Positive: $(-\infty, -5)\cup(1, \infty)$

Negative: $(-5, 1)$

Maximums / minimums: $(-2, -.8)$

Symmetry: none

End Behavior/Limits:

$\lim_{x\to\infty} f(x) = 0$ $\lim_{x\to 0^-} f(x) = 0$

$\lim_{x\to -5^-} f(x) = \infty$ $\lim_{x\to -5^+} f(x) = -\infty$

$\lim_{x\to 1^-} f(x) = -\infty$ $\lim_{x\to 1^+} f(x) = \infty$

Vertical Asymptote(s):

$x = -5$

$x = 1$

Horizontal Asymptote:

$y = 0$