

Date: 11/8/23

Section: 3.5

Objective: I can solve rational equations and inequalities

REVIEW: Simplify the following and fill in the blank:

a) $\frac{10}{0} = \text{undef}$

b) $\frac{0}{10} = 0$

c) Anything divided by zero is undef.

d) The denominator (bottom) of a fraction can't equal 0

ex

$$\frac{r+8}{r^2-6r-16} - \frac{5}{2r^2+4r}$$

$$\frac{(r+8)2r}{2r(r-8)(r+2)} + \frac{-5(r-8)}{2r(r+2)(r-8)}$$

$$\frac{2r^2+16r-5r+40}{2r(r-8)(r+2)}$$

$$\frac{2r^2+11r+40}{2r(r-8)(r+2)}$$

EXAMPLES: State the restrictions for each rational equation:

a) $\frac{5}{x+4} = 2$

$x \neq -4$

b) $\frac{4x}{8x-3} = \frac{7}{x}$

$x \neq \frac{3}{8}, 0$

c) $\frac{5x-2}{5} = \frac{2x}{5}$

no restrictions
linear

d) $\frac{x-2}{x^2} = \frac{1}{2x}$

$x \neq 0$

e) $\frac{x+9}{x^2+6x+8} = \frac{4x+1}{x-6}$
 $(x+4)(x+2)$

$x \neq -4, -2, 6$

f) $\frac{7x+4}{x^2+3x} = \frac{1}{x}$
 $x(x+3)$

$x \neq -3, 0$

Steps for Solving Rational Equations:

1. Factor the denominator and find Lowest Common Denominator (LCD).
2. Multiply the entire equations by the LCD to get rid of the fractions.
3. Solve for the variable.
4. State the restrictions and check against your answers.

EXAMPLES: State the restrictions. Solve the equation algebraically. Identify the extraneous solutions.
Show work!

$$1. \left(1 - \frac{1}{5x} = \frac{4}{5x} \right) \left(\frac{5x}{1} \right)$$

$$\begin{aligned} 5x - 1 &= 4 \\ 5x &= 5 \\ x &= 1 \\ x &\neq 0 \end{aligned}$$

$$2. \left(4 + \frac{6}{n-1} = \frac{1}{n-1} \right) \left(\frac{n-1}{1} \right)$$

$$\begin{aligned} 4n - 4 + 6 &= 1 \\ -2 & \quad -2 \\ 4n &= -1 \\ n &= -\frac{1}{4} \\ n &\neq 1 \end{aligned}$$

$$3. \left(\frac{1}{x+3} + \frac{x+2}{x^2+3x} = \frac{1}{x} \right) \left(\frac{x(x+3)}{1} \right)$$

$$\begin{aligned} x + x + 2 &= x + 3 \\ 2x + 2 &= x + 3 \\ x &= 1 \\ x &\neq 0, -3 \end{aligned}$$

$$4. \left(\frac{5}{x^2-7x+12} - \frac{2}{3-x} = \frac{5}{x-4} \right) \left(\frac{-(x-3)(x-4)}{1} \right)$$

$$\begin{aligned} -5 - 2x + 8 &= -5x + 15 \\ 3x &= 12 \end{aligned}$$

$$\begin{aligned} x &= 4 \text{ is extraneous} \\ x &\neq 4, 3 \end{aligned}$$

$$5. \frac{2}{x-3} + \frac{6}{x+5} = \frac{2}{x^2+2x-15}$$

$$6. \left(\frac{x-4}{x} - \frac{3}{x+1} + \frac{4}{x^2+x} = 0 \right) \left(\frac{x(x+1)}{1} \right)$$

$$x^2 - 3x - 4 - 3x + 4 = 0$$

$$x^2 - 6x = 0$$

$$x(x-6) = 0$$

$$x = \cancel{0}, 6$$

$$x \neq 0, -1$$

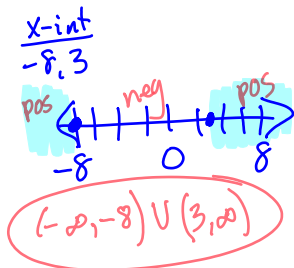
Steps for Solving Inequalities:

1. Set inequality $>$, $<$, \geq , or \leq to 0.
2. Factor the numerator and denominator.
3. Find the vertical asymptotes and the x -coordinate of any holes.
4. Find the x -intercepts.
5. Make a sign array to find where the graph is positive and negative.
6. Write the answer in interval notation.
 - $>$, \geq mean you want the positive sections
 - $<$, \leq mean you want the negative sections
 - $<$, $>$ mean the x -intercepts are not included so you put parentheses on the answer ()
 - \leq , \geq mean the x -intercepts are included so you put brackets on the answer []
 - Asymptotes and holes are ALWAYS parentheses!

EXAMPLES: Solve each inequality using sign charts. Write answers in interval notation.

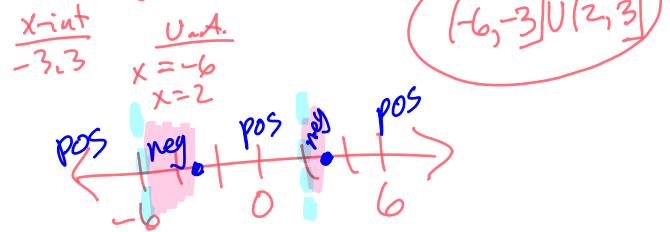
1. $x^2 + 5x - 24 > 0$

$$(x+8)(x-3) > 0$$

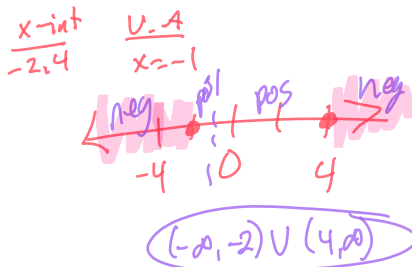


2. $\frac{x^2-9}{x^2+4x-12} \leq 0$

$$\frac{(x+3)(x-3)}{(x+6)(x-2)} \leq 0$$



3. $\frac{(x+2)(4-x)}{(x+1)^2} < 0$



4. $-2 < 3x - 1 \leq 8$ compound

$$+1 < 3x \leq 9$$

$$-\frac{1}{3} < x \leq 3$$

$$\left(-\frac{1}{3}, 3\right]$$

5. $(x+4)\sqrt{x+2} \geq 0$ restriction
 $x \geq -2$

