

SM 3H

DATE: 11/10/23

SECTION: 3.6

OBJECTIVE: I can find the partial decomposition of rational expressions.

If you are given $\frac{3}{4}$, what 2 fractions add to that fraction?

$$\frac{1}{4} + \frac{1}{2}$$

Decomposition of partial fractions:

given ans. find 2 fractions that add to answer

STEPS

1. Factor

2. use factors to write denom

ex $\frac{6x+7}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$

** if mult in factors: $\frac{3x}{(x+3)^3} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3}$

~~not~~ not factorable to linear: $\frac{3x-1}{(x^2+4)(x+1)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+1}$

3. mult by LCD to get rid of fractions

4. way 1 ~~DO NOT~~ distribute
use a restriction for x + solve

way 2 - distribute
make 2 eq + solve by system of eq.

5. way 1
use a dif restrictions or just pick a #

way 2
use ans. from step 4 to sub in + solve

6. write ans in correct notation

EXAMPLE: Write the terms of the partial fraction decomposition of the rational function. Do NOT solve for the constants. (only do steps 1-2)

$$1. \frac{7x+15}{x(x+5)} = \frac{A}{x} + \frac{B}{x+5}$$

$$2. \frac{-x^3+9x^2-20x+4}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

EXAMPLE: Find the partial fraction decomposition.

$$1. \frac{-2x-35}{(x+4)(x-5)} = \frac{A}{x+4} + \frac{B}{x-5}$$

$$-2x-35 = A(x-5) + B(x+4)$$

$$x=5 \quad -2(5)-35 = A(\cancel{5-5}) + B(5+4)$$

$$-45 = 9B$$

$$B = -5$$

$$x=-4 \quad -2(-4)-35 = A(-4-5) + B(\cancel{-4+4})$$

$$-27 = -9A$$

$$A = 3$$

$$\frac{3}{x+4} + \frac{-5}{x-5}$$

$$2. \frac{2x+1}{x^2-4x+4} = \frac{2x+1}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$$

$$2x+1 = Ax-2A+B$$

$$2x = Ax \quad 1 = -2A+B$$

$$A=2 \quad 1 = -2(2)+B$$

$$B=5$$

$$\frac{2}{x-2} + \frac{5}{(x-2)^2}$$

$$2x+1 = A(x-2) + B$$

$$x=2 \quad 2(2)+1 = A(\cancel{2-2}) + B$$

$$B=5$$

$$x=0 \quad 2(0)+1 = A(0-2) + 5$$

$$1 = -2A+5$$

$$-4 = -2A$$

$$A=2$$

$$3. \frac{2x}{x^2+5x+4} = \frac{2x}{(x+4)(x+1)} = \frac{A}{x+4} + \frac{B}{x+1}$$

$$2x = A(x+1) + B(x+4)$$

$$x=-1 \quad 2(-1) = A(\cancel{-1+1}) + B(-1+4)$$

$$-2 = 3B$$

$$B = -\frac{2}{3}$$

$$x=-4 \quad 2(-4) = A(-4+1) + B(\cancel{-4+4})$$

$$-8 = -3A$$

$$A = \frac{8}{3}$$

$$\frac{\frac{8}{3}}{x+4} + \frac{-\frac{2}{3}}{x-1}$$

$$4. \frac{4x^2}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$4x^2 = A(x-2)^2 + B(x-1)(x-2) + C(x-1)$$

$$x=1 \quad 4(1)^2 = A(1-2)^2 + B(\cancel{1-2}) + C(\cancel{1-1})$$

$$4 = A$$

$$x=2 \quad 4(2)^2 = A(\cancel{2-2})^2 + B(\cancel{2-2})(2-2) + C(2-1)$$

$$16 = C$$

$$x=0 \quad 4(0)^2 = 4(0-2)^2 + B(0-1)(0-2) + 16(0-1)$$

$$0 = 16 + 2B - 16$$

$$B=0$$

$$\frac{4}{x-1} + \frac{0}{x-2} + \frac{16}{(x-2)^2}$$

way 1

way 2

$$2x = Ax + A + Bx + 4B$$

$$2x = A\cancel{x} + B\cancel{x} - \text{ignore } x$$

$$0 = A + 4B \quad \leftarrow \text{plug in } 0 = A + 4\left(\frac{2}{3}\right)$$

$$0 = -A - 4B$$

$$\frac{0 = -3B}{2 = -3B} \quad B = -\frac{2}{3}$$

$$\frac{8}{3} = A$$