SM 3H

Date: 11/20/23 Section: 4.1

Objective: I can solve linear, quadratic, absolute value and radical equations.

Vocabulary and Review:

Linear: degree is one

Quadratic: degree is two

Cubic: degree is three

Absolute value: distance from 0, must have 2 ans. ±

Radicand: what's under radical (√)

Index: little # in √ tells you root to take

Exponent: little #, tells how many times to mult base

Square root: 1 of 2 same factors

Cube root: 1 of 3 same factors

Extraneous: solution not in domain

Solving an equation means finding solutions/numbers that makes the equation true.

Steps for solving an equation that only has 1 variable:

1. Isolate the parent function and variable.
2. Do the inverse of the parent function.

   **Remember if you take an even root, the answer must have ± on it.

3. Solve for the variable.
4. Check. Write if there are any extraneous answers or restrictions.

   **even roots— radicand ≥ 0

   **odd roots— none

never together

EXAMPLES: Solve for the variable, include both real and imaginary solutions. State the restrictions (domain). Write your solutions in simplest form.

1. $\frac{2}{3}x - \frac{1}{2} = \frac{1}{6}$

$4x - 3 = 1$

$4x = 4$

$x = 1$

2. $\frac{x-2}{3} = \frac{5}{9}$

$3x - 6 = 5$

$3x = 11$

$x = \frac{11}{3}$

3. $9x^2 - 20 = 5$

$9x^2 = 25$

$x^2 = \frac{25}{9}$

$x = \pm\frac{5}{3}$

4. $5x^3 + 2 = 42$

$5x^3 = 40$

$x^3 = 8$

$x^3 = 2$

5. $x^4 = 48$

$x = \pm 4\sqrt{2}$

6. $(x+4)^2 + 5 = 21$

$(x+4)^2 = 16$

$x + 4 = \pm 4$

$x = 8, 0$

7. $\sqrt{3x+1} - 4 = 0$

$\sqrt{3x+1} = 4$

$3x + 1 = 16$

$3x = 15$ $\quad 3x + 1 \geq 0$

$x = 5$ $\quad x \geq -\frac{1}{3}$

8. $5\sqrt{x} + 15 = -10$ $\quad x \geq 0$

$5\sqrt{x} = -25$

$\sqrt{x} = -5$

no solution

9. $3 - 2\sqrt[3]{x+1} = 5$

$-2\sqrt[3]{x+1} = 2$

$\sqrt[3]{x+1} = -1$

$x + 1 = -1$

$x = -2$

10. $4 - 5\sqrt[4]{x+1} = -6$

$-5\sqrt[4]{x+1} = -10$

$\sqrt[4]{x+1} = 2$

$x + 1 = 16$

$x = 15$

$x \geq 0$

11. $3\sqrt[5]{10x-7} = 9$

$\sqrt[5]{10x-7} = 3$

$10x - 7 = 243$

$10x = 250$

$x = 25$

12. $\sqrt{x-4} = (x-4)^2$

$x - 4 = x^2 - 8x + 16$

$x^2 - 9x + 20 = 0$

$(x-4)(x-5) = 0$

$x = 5, 4$

$x \geq 4$

Steps for solving an equation that only has more than 1 variable:

1. Set equation equal to 0.
2. Factor or do quadratic formula.
3. Set each factor equal to 0 or simplify the quadratic formula.
4. Check. Write if there are any extraneous answers.

EXAMPLES: Solve for the variable, include both real and imaginary solutions. State the restrictions (domain). Write your solutions in simplest form.

1. $4x^2 = 12x$

$4x^2 - 12x = 0$

$4x(x-3) = 0$

$x = 0, 3$

2. $x^2 + 8x = 10$

$x^2 + 8x - 10 = 0$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(1)(-10)}}{2(1)}$

$x = \frac{-8 \pm \sqrt{104}}{2}$

$x = \frac{-8 \pm 2\sqrt{26}}{2}$

$x = -4 \pm \sqrt{26}$

3. $3x^2 - 13x - 10 = 0$

$(3x+2)(x-5) = 0$

$x = -\frac{2}{3}, 5$

Steps for Solving Absolute Value Equations:

1. Get the absolute value alone on one side of the equation with a number on the other side.
2. Write **2** equations, one for the positive option and one for the negative option that gives you that answer. Solve.
3. If the number opposite the absolute value is negative, there is **no solution**.

1. $|x| = 8$

$x = \pm 8$

2. $\left|\frac{x}{5}\right| = 4$

$\frac{x}{5} = 4 \quad \frac{x}{5} = -4$

$x = \pm 20$

3. $|x-3| = 9$

$x - 3 = \pm 9$

$x = 12, -6$

4. $\frac{1}{3}|x-7| = 2$

$|x-7| = 6$

$x - 7 = 6 \quad x - 7 = -6$

$x = 13, 1$

5. $2|4x| - 8 = 16$

$2|4x| = 24$

$|4x| = 12$

$4x = \pm 12$

$x = \pm 3$

6. $6|5x-1| + 4 = 88$

$6|5x-1| = 84$

$|5x-1| = 14$

$5x - 1 = 14 \quad 5x - 1 = -14$

$5x = 15 \quad 5x = -13$

$x = 3 \quad x = -\frac{13}{5}$