SM 3H

Date: 11/27/23 Section: 4.2

Objective: I can solve rational exponents, specified variable, and "U" substitution.

REVIEW: rational exp

RULE: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ *and* $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

EXAMPLES: Write each expression in radical form.

1. $x^{\frac{1}{2}} = \sqrt[2]{x^1} = \sqrt{x}$
2. $(2x)^{\frac{4}{3}}$ $\sqrt[3]{2x}^{\,4}$ or $\sqrt[3]{(2x)^4}$ or $\sqrt[3]{16x^4}$
3. $(x+4)^{\frac{3}{4}}$ $\sqrt[4]{(x+4)^3}$

EXAMPLES: Write each expression in exponent form.

4. $\sqrt{x} = x^{\frac{1}{2}}$
5. $\sqrt[5]{(x-3)^7}$ $(x-3)^{\frac{7}{5}}$
6. $\sqrt[3]{(4x)^2} = (4x)^{\frac{2}{3}}$

RULE: $(x^m)^n = x^{mn}$

EXAMPLES: Simplify each expression.

7. $(x^3)^4 = x^{12}$
8. $\left(x^{\frac{1}{4}}\right)^{\frac{2}{5}} = x^{\frac{1}{10}}$
9. $\left(x^{\frac{1}{2}}\right)^2 = x$

EXAMPLES: Fill in the box to make the statement true.

10. $3 \cdot \boxed{\tfrac{1}{3}} = 1$
11. $\frac{1}{3} \cdot \boxed{3} = 1$

EXAMPLES: Fill in the box with the missing exponent that makes the statement true.

12. $\left(x^{\frac{1}{2}}\right)^{\boxed{2}} = x$
13. $\left((x+2)^{\frac{1}{5}}\right)^{\boxed{5}} = x+2$
14. $\left((4x)^{\frac{1}{3}}\right)^{\boxed{3}} = 4x$

15. $\left(\sqrt[3]{x}\right)^{\boxed{3}} = x$
16. $\left(\sqrt{5x}\right)^{\boxed{2}} = 5x$
17. $\left(\sqrt[4]{x-3}\right)^{\boxed{4}} = x-3$

Steps for solving an equation that only has 1 variable:

1. Isolate the parent function and variable.
2. Do the inverse of the parent function.

   **Remember if you take an even root, the answer must have ± on it. – if num to start is even

3. Solve for the variable.
4. Check. Write if there are any extraneous answers. – if denom to start is even

EXAMPLES: Solve for the variable, include both real and imaginary solutions. State the restrictions (domain). Write your solutions in simplest form.

1. $(x+7)^{\frac{2}{3}}+6=10$

$\sqrt[2]{4}^{3}$

$\left((x+7)^{\frac{2}{3}}=4\right)^{\frac{3}{2}}$

$x+7=\pm 8$

$x=+1,-15$

2. $5(x-4)^{\frac{3}{4}}-25=15$

$x-4\geq 0$

$x\geq 4$

$5(x-4)^{\frac{3}{4}}=40$

$\left((x-4)^{\frac{3}{4}}=8\right)^{\frac{4}{3}}$

$\sqrt[3]{8}^{4}$

$x-4=16$

$x=20$

3. $\frac{1}{2}(x+20)^{\frac{4}{3}}-5=3$

$\frac{1}{2}(x+20)^{\frac{4}{3}}=8$

$\left((x+20)^{\frac{4}{3}}=16\right)^{\frac{3}{4}}$

$x+20=\pm 8$

$x=-28,-12$

4. $2(x+1)^{\frac{2}{3}}+5=13$

$2(x+1)^{\frac{2}{3}}=8$

$\left((x+1)^{\frac{2}{3}}=4\right)^{\frac{3}{2}}$

$x+1=\pm 8$

$x=-9,7$

Steps for Solving for a Specified Variable:

1. Go backwards in the order of operations to get desired variable by itself.

Solve for the specified variable.

1. $y+10x=3$ (solve for x) $10x=3-y$ $x=\frac{3-y}{10}$ OR $x=\frac{3}{10}-\frac{y}{10}$	2. $16+2y=18x-2$ (solve for y) $2y=18x-18$ $y=9x-9$	3. $I=\frac{E}{R}$ (solve for R) $\frac{1}{I}=\frac{R}{E}$ $\frac{E}{I}=R$

4. $ax - by = c$ (solve for x)	5. $y = \dfrac{x-3}{x+2}$ (solve for x) $xy + 2y = x - 3$ ① mult by denom ② move together $xy - x = -3 - 2y$ ③ factor $x(y-1) = -3-2y$ $x = \dfrac{-3-2y}{y-1}$ or $x = \dfrac{3+2y}{-y+1}$	6. $\sqrt{d^2 - 4ef} = g$ (solve for d) $d^2 - 4ef = g^2$ $d^2 = g^2 + 4ef$ $d = \pm\sqrt{g^2 + 4ef}$

An equation is *quadratic in form* if it can be written as $au^2 + bu + c = 0$ **where** a $\neq 0$ **and *u* is a variable expression.**

Steps to Solve Equations Quadratic in Form:

1. Check to see if the equation is quadratic in form. If it is quadratic in form, the equation will have two variable expressions, and one will be the square of the other.
2. If the equation is quadratic in form, use "*u*-substitution." Let *u* = variable expression of the 2nd term.
3. Solve the equation for "*u*" using quadratic methods (factoring, square root principle, completing the square, quadratic formula).
4. Plug the substitution back in for *u* and solve for the original variable. (Don't forget the ± if you take an even root of both sides).
5. Check for extraneous solutions. (If you raise both sides of the equation to an even power, there may be extraneous solutions.)

Examples:

a) $x^4 - 10x^2 + 25 = 0$

$u = x^2$

$u^2 = x^4$

$u^2 - 10u + 25 = 0$

$(u-5)(u-5) = 0$

$u = 5$

$x^2 = 5$

$x = \pm\sqrt{5}$

b) $2y^6 - 7y^3 + 3 = 0$

$u = y^3$

$2u^2 - 7u + 3 = 0$

$(2u-1)(u-3) = 0$

$u = \frac{1}{2}$ $\quad u = 3$

$y^3 = \frac{1}{2}$ $\quad y^3 = 3$

$y = \frac{1}{\sqrt[3]{2}}, \sqrt[3]{3}$

c) $(2z+5)^2 - 4(2z+5) = 1$

$2z+5 = u$

$(2z+5)^2 - 4(2z+5) - 1 = 0$

$u^2 - 4u - 1 = 0$

$u = \dfrac{+4 \pm \sqrt{(-4)^2 - 4(-1)(1)}}{2(1)}$

$u = \dfrac{4 \pm \sqrt{20}}{2}$

$u = \dfrac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

$2z + 5 = 2 \pm \sqrt{5}$

$2z = -3 \pm \sqrt{5}$

$z = \dfrac{-3 \pm \sqrt{5}}{2}$

d) $(x^2 - 1)^2 - (x^2 - 1) - 2 = 0$

$u = x^2 - 1$

$u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$u = 2 \quad u = -1$

$x^2 - 1 = 2 \quad x^2 - 1 = -1$

$x^2 = 1 \quad x^2 = 0$

$x = \pm 1 \quad x = 0$

e) $n - 3\sqrt{n} - 4 = 0$

$u = \sqrt{n}$

$n \geq 0$

$n - 3n^{\frac{1}{2}} - 4 = 0$

$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$

$\sqrt{n} = 4 \quad \cancel{\sqrt{n} = -1}$

$n = 16$

f) $x^{2/3} + 6x^{1/3} + 8 = 0$

$u = x^{\frac{1}{3}}$

$u^2 + 6u + 8 = 0$

$(u+4)(u+2) = 0$

$u = -4 \quad u = -2$

$x^{\frac{1}{3}} = -4 \quad x^{\frac{1}{3}} = -2$

$x = -64, -8$

g) $2m^{-2} + m^{-1} = 15$

$u = m^{-1}$ or $\frac{1}{m}$

$m \neq 0$

$2u^2 + u - 15 = 0$

$(2u-5)(u+3) = 0$

$u = \frac{5}{2} \quad u = -3$

$\frac{1}{m} = \frac{5}{2} \quad \frac{1}{m} = -3$

$m = \frac{2}{5} \quad m = -\frac{1}{3}$