SM 3H

Date: 12/1/23 **Section:** 4.4

Objective: I can simplify a complex fraction.

Complex Fractions

A complex fraction is when there is more than one rational fraction within a rational fraction.

$\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$

$\frac{\left(1+\frac{1}{2}\right)}{3}$

To simplify a complex fraction:

1. Factor denominators
2. Find LCD for top fraction and LCD for bottom fraction
3. Multiply to get LCD
4. Add numerators of both fractions
5. Do stay change flip
6. Factor
7. Cross off ones
8. Write what's left

EX. Simplify.

a. $\frac{\frac{2x}{11}}{\frac{5x}{11}}$ $\frac{2x}{5x}$ $\frac{2}{5}$

$\frac{2x}{x} = \frac{2}{5}$

b. $\frac{\frac{1}{4}-\frac{3}{16}}{\frac{1}{3}+\frac{1}{2}}$ $\frac{1}{16} = \frac{1}{16} \cdot \frac{8}{5} = \frac{3}{40}$

c. $\frac{\frac{3x}{4y^2}}{\frac{2x}{y}}$ $\frac{3x}{4y^2} \cdot \frac{y}{2x} = \frac{3}{8y}$

d. $\frac{3x-\frac{5x}{2x-3}}{3x+\frac{5x}{2x+3}}$ $\frac{6x^2-9x-5x}{2x-3} = \frac{6x^2-14x}{2x-3}$, $\frac{2x+3}{6x^2+14x}$

$\frac{2x(3x-7)(2x+3)}{(2x-3)(2x)(3x+7)}$

$\frac{(3x-7)(2x+3)}{(2x-3)(3x+7)}$

e. $\frac{\frac{2}{x}+\frac{2}{4x^2-1}}{2x+\frac{1}{2x-1}}$ $\frac{\frac{2(2x-1)(2x+1)}{x(2x-1)(2x+1)} + \frac{2x}{(2x-1)(2x+1)x}}{\frac{2x(2x-1)}{2x-1} + \frac{1}{2x-1}}$

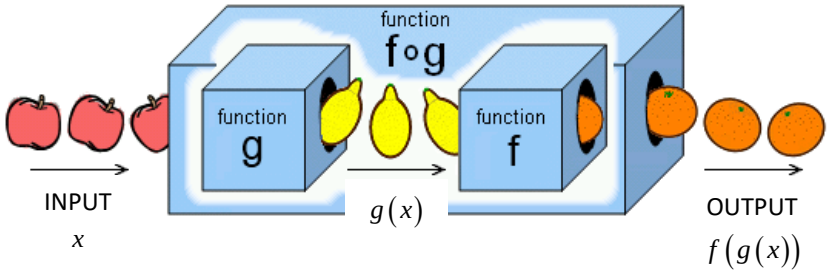
$\frac{8x^2-2+2x}{x(2x-1)(2x+1)}$

$\frac{4x^2-2x+1}{2x-1}$

$\frac{8x^2+2x-2}{x(2x+1)(4x^2-2x+1)}$

f. $\frac{2-\frac{4}{x^2}}{x-2}$

Composite Function: In a composite function, one function is performed, and then a second function is performed on the result of the first function. $(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$.



Hints:

- Work inside out. Plug the input into the inside function, then plug the result into the outside function.
- $(f \circ g)(x) = f(g(x))$ is not the same as $(f \cdot g)(x) = f(x) \cdot g(x)$.

Composition of functions — Multiplication of functions

Example: Evaluate each expression using the values given in the table.

x	−3	−2	−1	0	1	2	3
$f(x)$	−7	−5	−3	−1	3	5	7
$g(x)$	8	3	0	−1	0	3	8

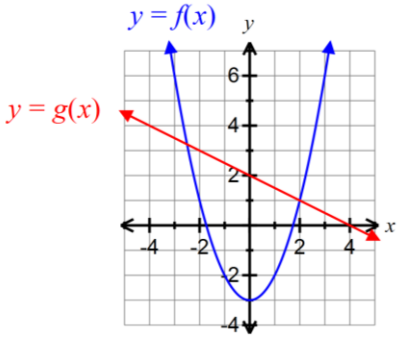
a) $(f \circ g)(-2)$ $f(g(-2)) = f(3) = 7$

b) $(g \circ f)(-1)$ $g(-3) = 8$

c) $(f \circ f)(1) = 7$

d) $(g \circ g)(0)$

Example: Evaluate each expression using the graph.



a) $(f \circ g)(4)$ $f(g(4)) = f(0) = 3$

b) $(g \circ f)(-1)$ $g(-2) = 3$

c) $(f \circ f)(1)$

d) $(g \circ g)(0) = 1$

Example: $f(x) = 2x^2$ and $g(x) = 1 - 3x^2$

a) Find $(f \circ g)(4)$ $2(1-3(4)^2)^2 = 4418$

$g(4) = 1-3(4)^2 = -47$ $f(-47) = 2(-47)^2$

b) Find $(g \circ f)(2)$ $1-3(2(2)^2)^2 = -191$

c) Find $(f \circ f)(1)$

d) Find $(g \circ g)(0)$ $1-3(1-3(0)^2)^2 = -2$

Domain of a Composite Function

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

Example: Find the domain of the composite function $f \circ g$.

a) $f(x)=\dfrac{5}{x+4}$, $g(x)=\dfrac{8}{x}$

$g: (-\infty,0)\cup(0,\infty)$

$\dfrac{5}{\left(\frac{8}{x}\right)+4}$

$\dfrac{5}{\frac{4x+8}{x}} = 5\cdot\dfrac{x}{4x+8} = \dfrac{5x}{4x+8}$

$(-\infty,-2)\cup(-2,\infty)$

$D: (-\infty,-2)\cup(-2,0)\cup(0,\infty)$

b) $f(x)=\dfrac{x}{x-1}$, $g(x)=\dfrac{x+5}{x-4}$ $\quad x\neq 4$

$\dfrac{\left(\frac{x+5}{x-4}\right)}{\left(\frac{x+5}{x-4}\right)-1\left(\frac{x-4}{x-4}\right)}$

$\dfrac{\frac{x+5}{x-4}}{\frac{9}{x-4}} = \dfrac{x+5}{9}$

$D: (-\infty,4)\cup(4,\infty)$

Example: $f(x)=x+1$ and $g(x)=x^2+4$

a) Find $(f\circ g)(x)$ and its domain.

$(x^2+4)+1$

x^2+5

$(-\infty,\infty)$

b) Find $(g\circ f)(x)$ and its domain.

$(x+1)^2+4$

$D=(-\infty,\infty)$

Example: $f(x)=\dfrac{1}{x+3}$ and $g(x)=-\dfrac{2}{x}$

a) Find $(f\circ g)(x)$ and its domain.

b) Find $(g\circ f)(x)$ and its domain.

c) Find $(f \circ f)(x)$ and its domain.

d) Find $(g \circ g)(x)$ and its domain.

prove = must show ALL work

Example: Show that $(f \circ g)(x) = (g \circ f)(x) = x$.

a) $f(x) = 4x;\ g(x) = x/4$

$4\left(\frac{x}{4}\right) = x$ $\quad \frac{(4x)}{4} = x$

b) $f(x) = 4 - 3x;\ g(x) = \frac{1}{3}(4 - x)$

$4 - 3\left(\frac{1}{3}(4-x)\right)$

$4 - 3\left(\frac{4}{3} - \frac{x}{3}\right)$

$4 - 4 + x$

x $\quad =\!=\!=$ $\quad x$

$\frac{1}{3}(4 - (4 - 3x))$

$\frac{1}{3}(4 - 4 + 3x)$

$\frac{1}{3}(3x)$

x

Example: Find functions f and g such that $f \circ g = H$.

a) $H(x) = (x^2 + 1)^4$

$f(x) = x^4$

$g(x) = x^2 + 1$

$(f \circ g)(x)$

$f(x) = (x+1)^4$

$g(x) = x^2$

b) $H(x) = |2x + 1|$

$f(x) = |x|$

$g(x) = 2x + 1$