SM 3H

Date: **Section:**

Objective:

Laws of Exponents: If s, t, a, and b are real numbers with $a > 0$ and $b > 0$, then

$$a^s \cdot a^t = a^{s+t} \qquad \left(a^s\right)^t = a^{st} \qquad (ab)^s = a^s b^s \qquad 1^s = 1 \qquad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \qquad a^0 = 1$$

An ***exponential function*** is a function of the form____________ where a is a positive real number $(a > 0)$ and $a \neq 1$. The domain of f is the set of all real numbers.

Theorem: For an exponential function $f(x) = a^x,\ a > 0,\ a \neq 1,$ if x is any real number, then

$$a =$$

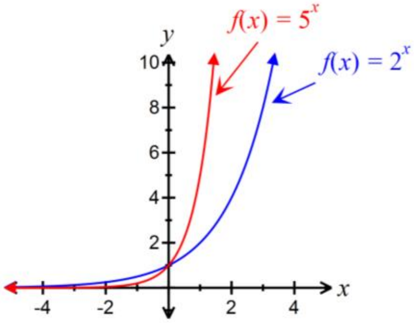
Examples: Determine whether the given functions are exponential or not.

x	$f(x)$
-1	2
0	5
1	8
2	11
3	14

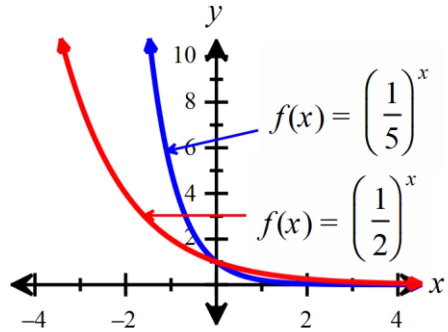
x	$f(x)$
-1	2/3
0	1
1	3/2
2	9/4
3	27/8

Properties of the Exponential Function $f(x) = a^x,\ a > 0,\ a \neq 1$

- Domain: _______________Range: __________________
- There are no________________; the y-intercept is __________.
- The x-axis $(y = 0)$ is a __________________ _______________________.
  - For $a > 1,$ the graph approaches the x-axis as ________________.
  - For $0 < a < 1,$ the graph approaches the x-axis as ________________.
- $f(x) = a^x$ is one-to-one.
  - For $a > 1,$ $f(x) = a^x$ is an __________________ function.
  - For $0 < a < 1,$ $f(x) = a^x$ is a __________________ function.
- The graph of f contains the points ___________, ______________, and ________________.
- The graph of f is smooth and continuous, with no corners, gaps, or cusps.



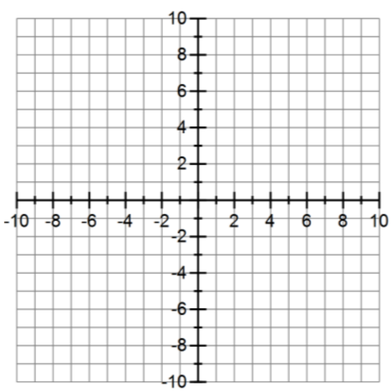
$a > 1$



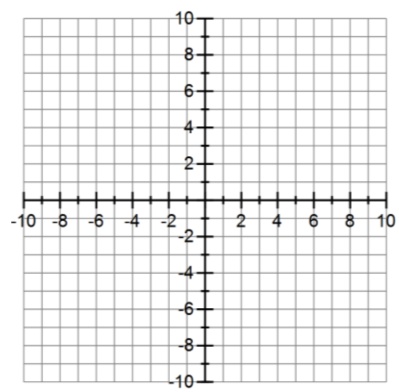
$0 < a < 1$

Examples:

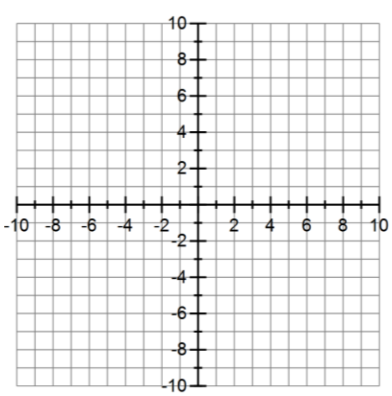
a) Graph $f(x) = 3^x$.



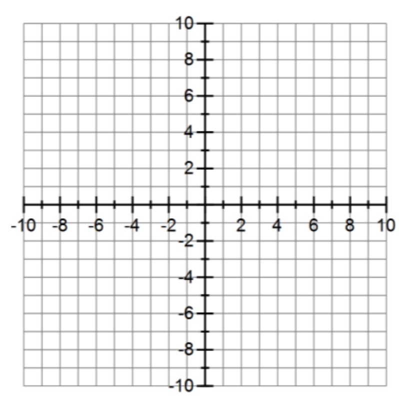
b) Graph $f(x) = 2 \cdot \left(\frac{1}{3}\right)^x$.



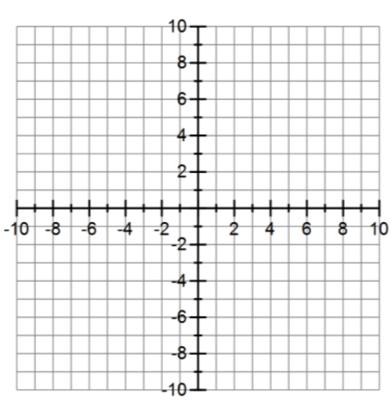
c) Graph $f(x) = 5^{x+3}$.



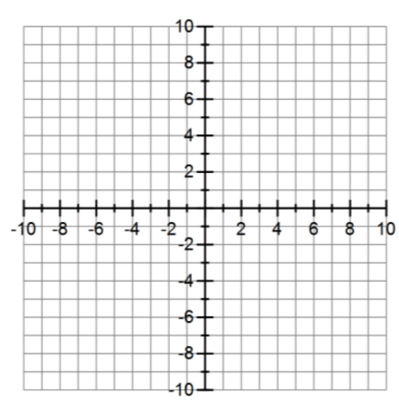
d) Graph $f(x) = \left(\frac{1}{2}\right)^x + 3$.



e) Graph $f(x) = 2^{-x}$.



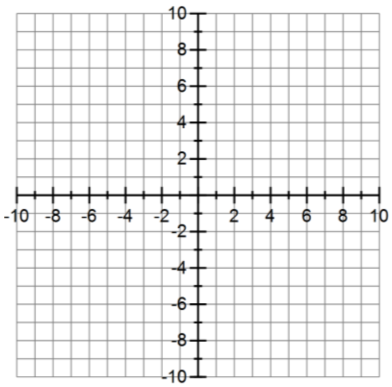
f) Graph $f(x) = -3^x$



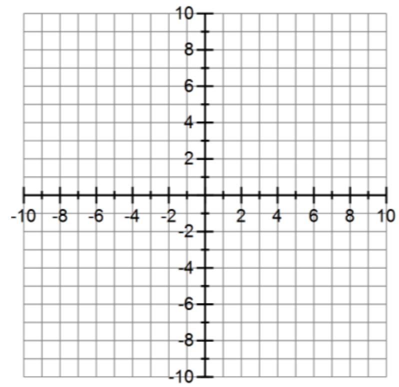
The **number *e*** (approximately 2.71828…) is defined as the number that the expression $\left(1+\frac{1}{n}\right)^n$ approaches as $n \to \infty$. In calculus, this is expressed using limit notation as $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.

Examples:

a) Graph $f(x) = e^x$.

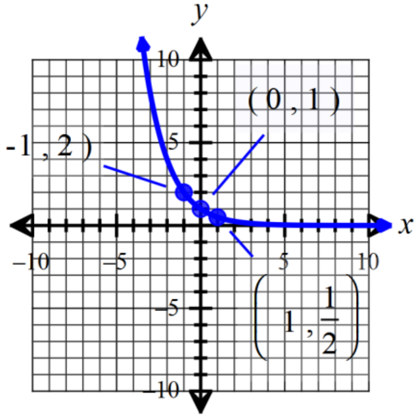


b) Graph $f(x) = -e^{-x}$

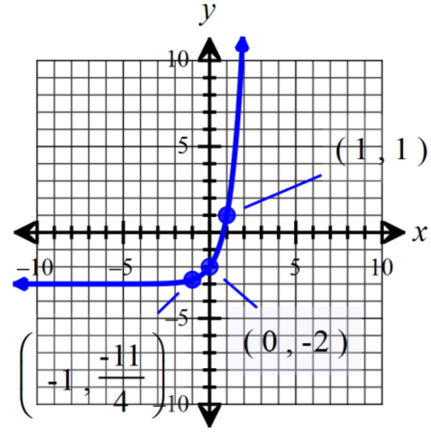


Determine the exponential function of the given graph.

a)



b)



Solving Exponential Equations

If $a > 0$ and $a \neq 1$ and $a^u = a^v$, then $u = v$.

Many exponential equations can be rewritten so the two sides have a common base. This allows us to set the exponents equal to each other and solve the equation.

Examples: Solve the following equations.

a) $3^{-x} = 243$

b) $5^{x+3} = \frac{1}{5}$

c) $27^{x^2} = 3^x$

d) $3^{x^2-5x} = \frac{1}{81}$

e) $3^x \cdot 9^{x^2} = 27^2$

f) $e^{x^2} = e^{3x} \cdot \frac{1}{e^2}$

When the interest due at the end of a payment period is added to the principal so that the interest computed at the end of the next payment period is based on the new principal amount (old principal plus interest), the interest is said to have been ***compounded***. ***Compound interest*** is interest paid on principal and previously earned interest.

Compound Interest Formula

The amount A after t years due to a principal P invested at an annual interest rate t compounded n times per year is $\boldsymbol{A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}}$.

Example: Investing \$1000 at an annual rate of 9% compounded annually, semiannually, quarterly, monthly, and daily will yield the following amounts after 1 year:

Annual Compounding (n = 1):

Semiannual Compounding (n = 2):

Quarterly Compounding (n = 4):

Monthly Compounding (n = 12):

Daily Compounding (n = 365):

Present Value**:** The amount of money that must be invested now in order to end up with a given amount after a certain amount of time.

The present value P of A dollars to be received after t years, assuming a per annum interest rate r is compounded n times per year, is $\boldsymbol{P = A \cdot \left(1 + \frac{r}{n}\right)^{-nt}}$.

Example: How much money must be invested now in order to end up with \$20,000 in 10 years at

a) 5% compounded quarterly?