

Date: 12/13/23 Section: 5.1

Objective: I can solve and graph exponential functions.

**Laws of Exponents:** If  $s$ ,  $t$ ,  $a$ , and  $b$  are real numbers with  $a > 0$  and  $b > 0$ , then

$$a^s \cdot a^t = a^{s+t} \quad (a^s)^t = a^{st} \quad (ab)^s = a^s b^s \quad 1^s = 1 \quad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \quad a^0 = 1$$

An **exponential function** is a function of the form  $a^x$  where  $a$  is a positive real number ( $a > 0$ ) and  $a \neq 1$ . The domain of  $f$  is the set of all real numbers.

**Theorem:** For an exponential function  $f(x) = a^x$ ,  $a > 0$ ,  $a \neq 1$ , if  $x$  is any real number, then

$$a =$$

**Examples:** Determine whether the given functions are exponential or not.

| $x$ | $f(x)$ |
|-----|--------|
| -1  | 2      |
| 0   | 5      |
| 1   | 8      |
| 2   | 11     |
| 3   | 14     |

+3 each time  
linear

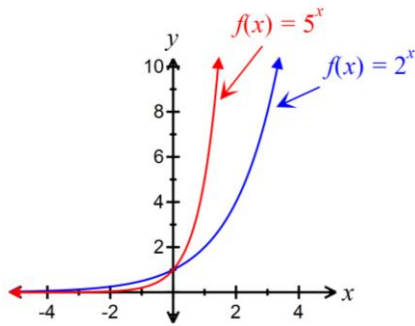
| $x$ | $f(x)$         |
|-----|----------------|
| -1  | $\frac{2}{3}$  |
| 0   | 1              |
| 1   | $\frac{3}{2}$  |
| 2   | $\frac{9}{4}$  |
| 3   | $\frac{27}{8}$ |

mult by  $\frac{3}{2}$   
exponential

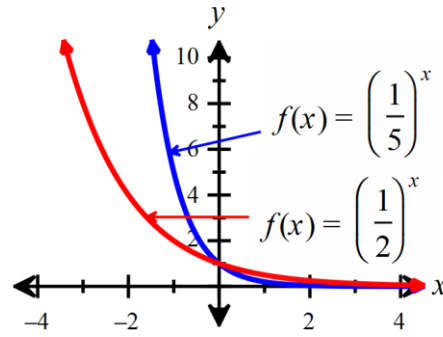
**Properties of the Exponential Function  $f(x) = a^x$ ,  $a > 0$ ,  $a \neq 1$**

- Domain: \_\_\_\_\_ Range: \_\_\_\_\_
- There are no \_\_\_\_\_; the y-intercept is \_\_\_\_\_.
- The x-axis ( $y = 0$ ) is a horizontal asymptote.
  - For  $a > 1$ , the graph approaches the x-axis as  $y \rightarrow 0$  left.
  - For  $0 < a < 1$ , the graph approaches the x-axis as  $y \rightarrow 0$  right.
- $f(x) = a^x$  is one-to-one.
  - For  $a > 1$ ,  $f(x) = a^x$  is an increasing function.
  - For  $0 < a < 1$ ,  $f(x) = a^x$  is a decreasing function.
- The graph of  $f$  contains the points  $(-1, \frac{1}{a})$ ,  $(0, 1)$ , and  $(1, a)$ .
- The graph of  $f$  is smooth and continuous, with no corners, gaps, or cusps.

$$\frac{1}{2} = 2^{-1}$$



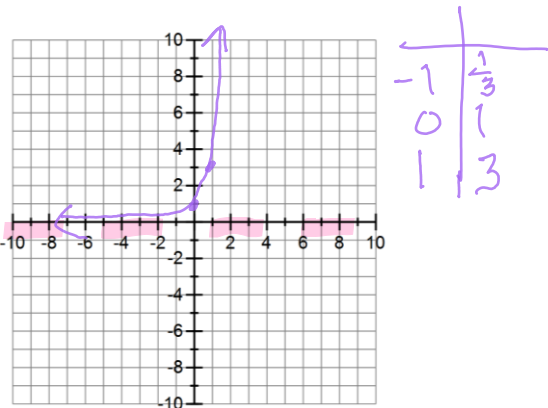
$a > 1$



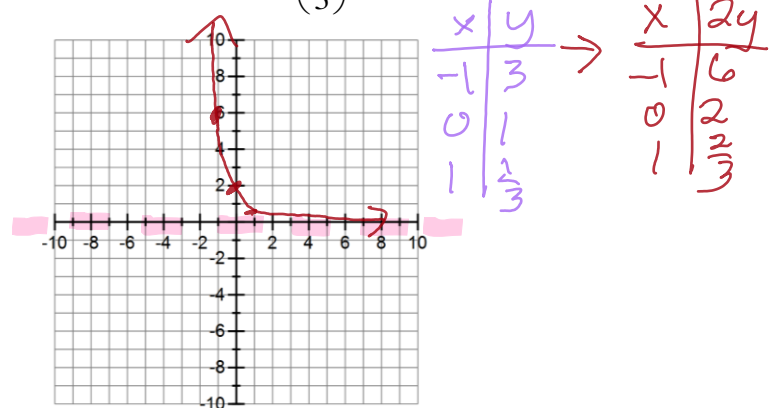
$0 < a < 1$

**Examples:**

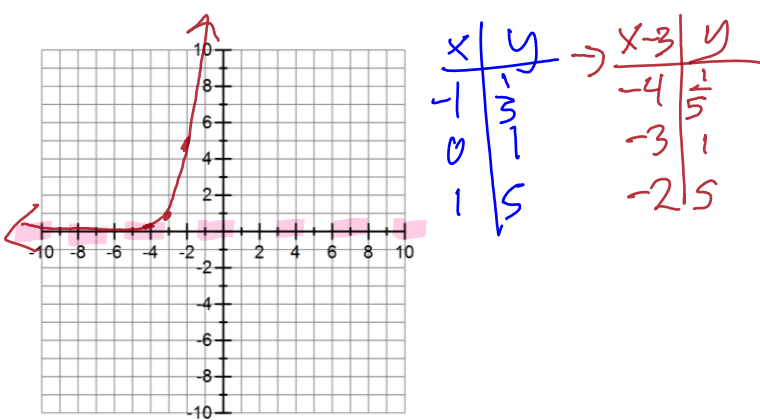
a) Graph  $f(x) = 3^x$ . *parent of base 3*



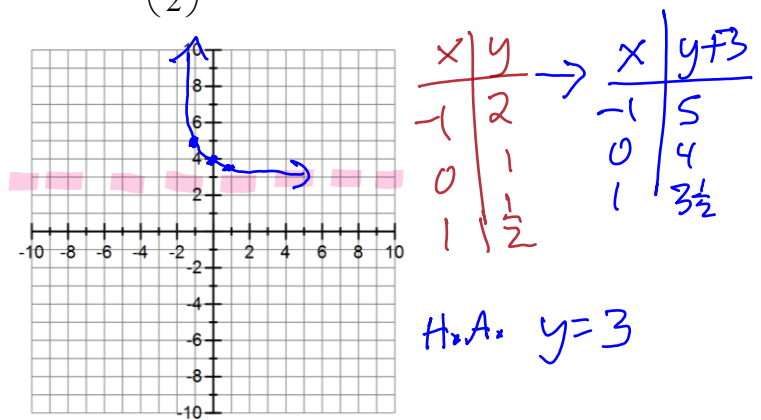
b) Graph  $f(x) = 2 \cdot \left(\frac{1}{3}\right)^x$ . *vert stretch of 2*



c) Graph  $f(x) = 5^{x+3}$ . *translate left 3*

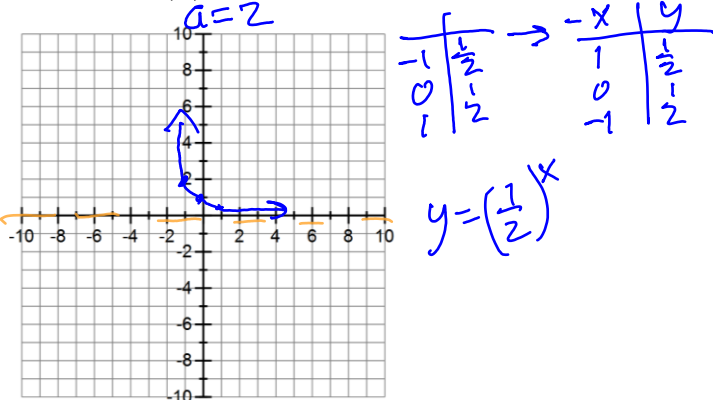


d) Graph  $f(x) = \left(\frac{1}{2}\right)^x + 3$ . *translate up 3*



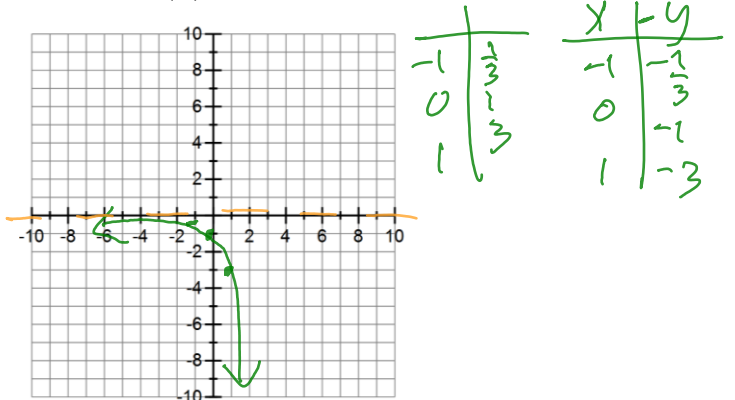
*H.A. y = 3*

e) Graph  $f(x) = 2^{-x}$ . *reflect over y-axis*



$y = \left(\frac{1}{2}\right)^x$

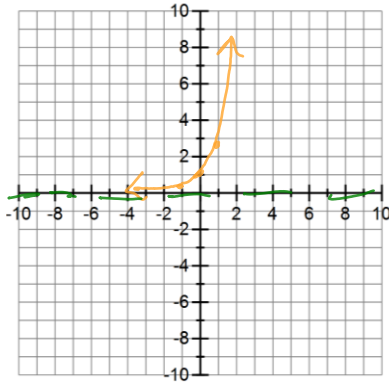
f) Graph  $f(x) = -3^x$ . *reflect over x-axis*



The **number e** (approximately 2.71828...) is defined as the number that the expression  $(1 + \frac{1}{n})^n$  approaches as  $n \rightarrow \infty$ . In calculus, this is expressed using limit notation as  $e = \lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n$ .

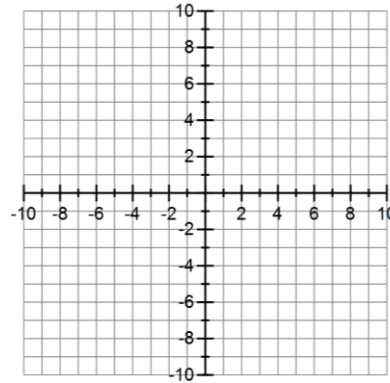
**Examples:**

a) Graph  $f(x) = e^x$ .



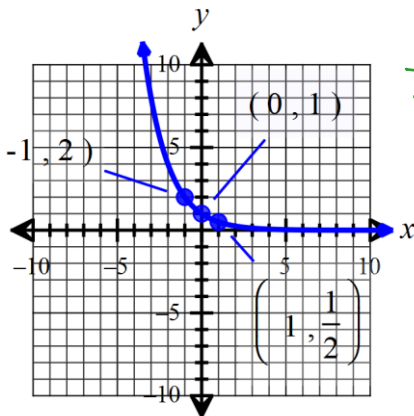
$$\begin{array}{r} -1 \mid \frac{1}{e} \approx 0.4 \\ 0 \mid 1 \\ 1 \mid e \approx 2.7 \end{array}$$

b) Graph  $f(x) = -e^{-x}$ .



**Determine the exponential function of the given graph.**

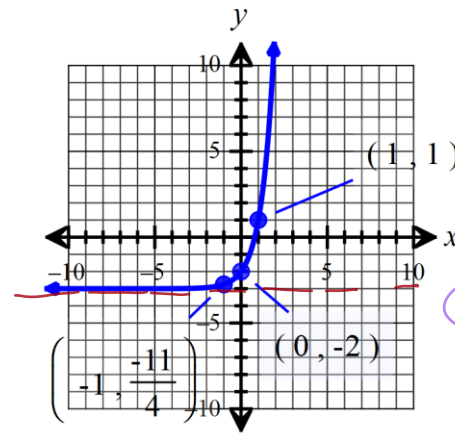
a)



$$\begin{array}{r} -1 \mid 2 \\ 0 \mid 1 \\ 1 \mid \frac{1}{2} \end{array}$$

$(\frac{1}{2})^x = f(x)$   
 $2^{-x} = f(x)$

b)



$$\begin{array}{r} -1 \mid -\frac{11}{4} = -2\frac{3}{4} + 3\frac{1}{4} \\ 0 \mid -2 + 3 = 1 \\ 1 \mid 1 + 3 = 4 \end{array}$$

$4^x - 3 = f(x)$

**Solving Exponential Equations**

If  $a > 0$  and  $a \neq 1$  and  $a^u = a^v$ , then  $u = v$ .

Many exponential equations can be rewritten so the two sides have a common base. This allows us to set the exponents equal to each other and solve the equation.

**Examples:** Solve the following equations.

a)  $3^{-x} = 243$       $243 = 3^5$   
 $3^{-x} = 3^5$   
 $-x = 5$       $x = -5$

b)  $5^{x+3} = \frac{1}{5}$   
 $5^{x+3} = 5^{-1}$   
 $x+3 = -1$       $x = -4$

c)  $27x^2 = 3^x$       $3^3 = 27$   
 $(3^3)^{x^2} = 3^x$   
 $3x^2 = x$   
 $3x^2 - x = 0$   
 $x(3x-1) = 0$   
 $x = 0, \frac{1}{3}$

d)  $3^{x^2-5x} = \frac{1}{81}$       $81 = 3^4$   
 $3^{x^2-5x} = 3^{-4}$   
 $x^2 - 5x = -4$   
 $x^2 - 5x + 4 = 0$   
 $(x-4)(x-1) = 0$   
 $x = 4, 1$

e)  $3^x \cdot 9^{x^2} = 27^2$   
 $3^x \cdot (3^2)^{x^2} = (3^3)^2$   
 $3^{x^2+x} = 3^6$   
 $x^2 + x = 6$   
 $x^2 + x - 6 = 0$   
 $(x+3)(x-2) = 0$   
 $x = -3, 2$

f)  $e^{x^2} = e^{3x} \cdot \frac{1}{e^2}$   
 $e^{x^2} = e^{3x-2}$   
 $x^2 = 3x-2$   
 $x^2 - 3x + 2 = 0$   
 $(x-1)(x-2) = 0$   
 $x = 1, 2$

When the interest due at the end of a payment period is added to the principal so that the interest computed at the end of the next payment period is based on the new principal amount (old principal plus interest), the interest is said to have been **compounded**. **Compound interest** is interest paid on principal and previously earned interest.

**Compound Interest Formula**

The amount  $A$  after  $t$  years due to a principal  $P$  invested at an annual interest rate  $r$  compounded  $n$  times per

year is  $A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$ .

*Handwritten notes:*  $= P$ ,  $= .09$ ,  $= n=1$ ,  $= n=2$ ,  $n=4$ ,  $n=12$

**Example:** Investing \$1000 at an annual rate of 9% compounded annually, semiannually, quarterly, monthly, and daily will yield the following amounts after 1 year:  $t=1$

Annual Compounding ( $n = 1$ ):

$1000 \left(1 + \frac{.09}{1}\right)^{1 \cdot 1} = \$1090$

Semiannual Compounding ( $n = 2$ ):

$1000 \left(1 + \frac{.09}{2}\right)^{2 \cdot 1} \approx \$1092.03$

Quarterly Compounding ( $n = 4$ ):

Monthly Compounding ( $n = 12$ ):

Daily Compounding ( $n = 365$ ):

**Present Value:** The amount of money that must be invested now in order to end up with a given amount after a certain amount of time.

The present value  $P$  of  $A$  dollars to be received after  $t$  years, assuming a per annum interest rate  $r$  is

compounded  $n$  times per year, is  $P = A \cdot \left(1 + \frac{r}{n}\right)^{-nt}$ .

**Example:** How much money must be invested now in order to end up with \$20,000 in 10 years at a) 5% compounded quarterly?

$$20000 = P \left(1 + \frac{.05}{4}\right)^{4 \cdot 10}$$


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$$\left(1 + \frac{.05}{4}\right)^{40}$$

$$P \approx \$12168.27$$