SM 3H

2023-2024

5.2 Logarithmic Functions

SCORE: /

Name _____ **Date** _____ **Period** _____

1. The domain of a logarithmic function $f(x) = \log_a x$ is _____.

2. The graph of every logarithmic function $f(x) = \log_a x$, where $a > 0$, and $a \neq 1$, passes through three points:

_____ , _____ , and _____ .

3. If the graph of a logarithmic function $f(x) = \log_a x$, where $a > 0$, and $a \neq 1$, is increasing, then its base must be larger than _____.

4. **True or False**: If $y = \log_a x$, then $y = a^x$.

5. **True or False:** The graph of $f(x) = \log_a x$, where $a > 0$, and $a \neq 1$, has an x-intercept equal to 1 and no y-intercept.

Change each exponential statement into an equivalent statement involving a logarithm.

6. $9 = 3^2$ 7. $a^2 = 1.6$ 8. $3^x = 4.6$ 9. $e^{2.2} = M$

Change each logarithmic statement to an equivalent statement involving an exponent.

10. $\log_2 8 = 3$ 11. $\log_a 3 = 6$ 12. $\log_2 6 = x$ 13. $\ln x = 4$

Find the exact value of each logarithm without using a calculator.

14. $\log_2 1$ 15. $\log_{1/2} 16$ 16. $\log_{\sqrt{2}} 4$ 17. $\ln \sqrt{e}$

Find the domain of each function.

18. $f(x)=\ln(x-3)$

19. $f(x)=3-2\log_4\left[\frac{x}{2}-5\right]$

20. $g(x)=\log_5\left(\frac{x+1}{x}\right)$

21. $g(x)=\frac{1}{\ln x}$

Use a calculator to evaluate each expression. Round your answer to three decimal places.

22. $\log 9.43$

23. $\log(-14)$

24. $\ln 4.05$

25. $\ln(-0.49)$

26. $\frac{\ln 5}{3}$

27. $\frac{\ln 4+\ln 2}{\log 4+\log 2}$

28. $\frac{2\ln 5+\log 50}{\log 4-\ln 2}$

Use the given function *f* to:

(*a*) Find the domain of *f* . (b) Find the vertical asymptote of *f*. (c) Write the transformations.

(d) Graph *f*. (e) From the graph determine the range.

Use transformations and a table of values for at least 5 key points to get the graphs. <u>No graphing calculators!</u>

29. $f(x)=3\ln(x+4)$

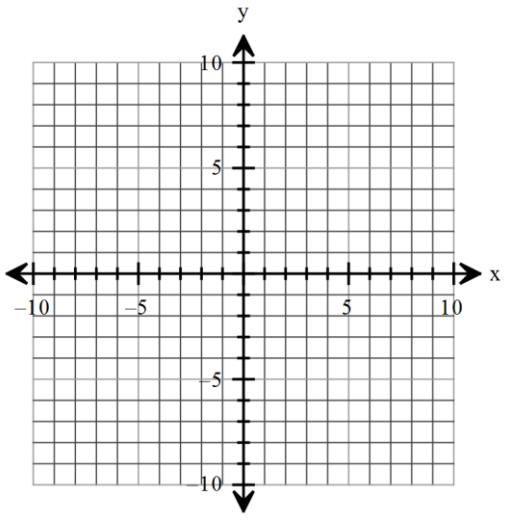
Domain: _____

Asymptotes:_____

Key points and transformations:

x	$f(x)$

x	$f(x)$



Range: _____

30. $f(x) = \log(-x) + 3$

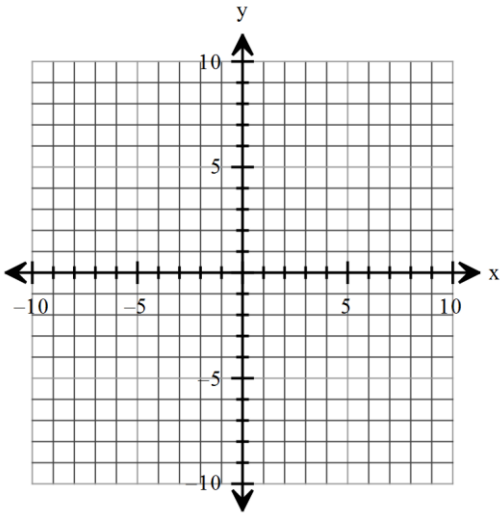
Domain: _____

Asymptotes:_____

Key points and transformations:

x	$f(x)$

x	$f(x)$



Range: _____

31. $f(x) = \ln(2 - x)$

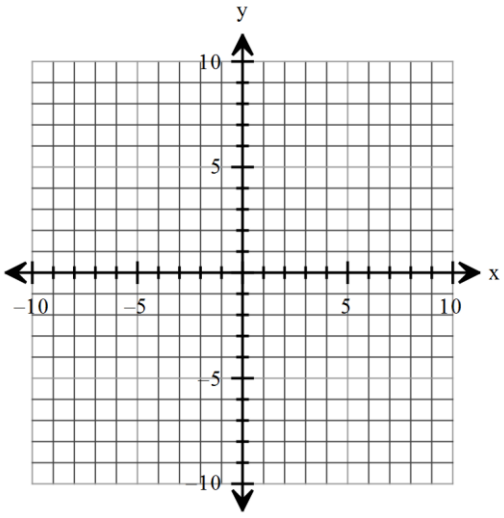
Domain: _____

Asymptotes:_____

Key points and transformations:

x	$f(x)$

x	$f(x)$



Range: _____

32. $f(x) = \log_3(2x + 5)$

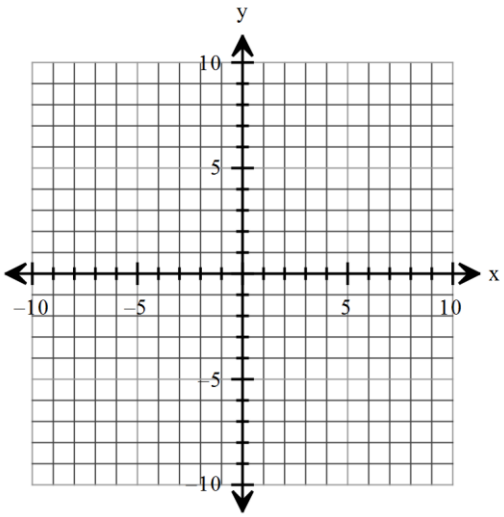
Domain: _____

Asymptotes:_____

Key points and transformations:

x	$f(x)$

x	$f(x)$



Range: _____

33. $f(x)=\log_3(x-4)+2$

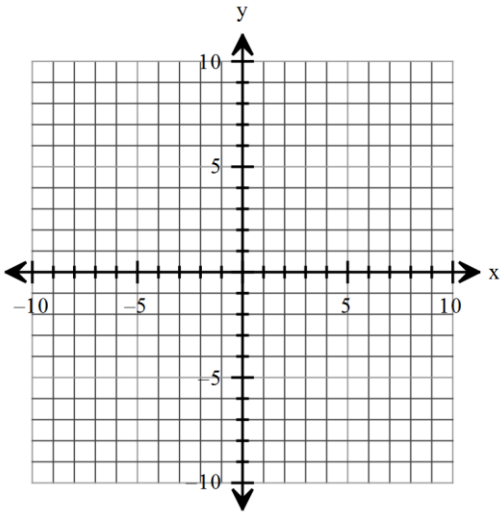
Domain: _____

Asymptotes:_____

Key points and transformations:

x	$f(x)$

x	$f(x)$



Range: _____

34. $f(x)=-2\log_3(x-5)$

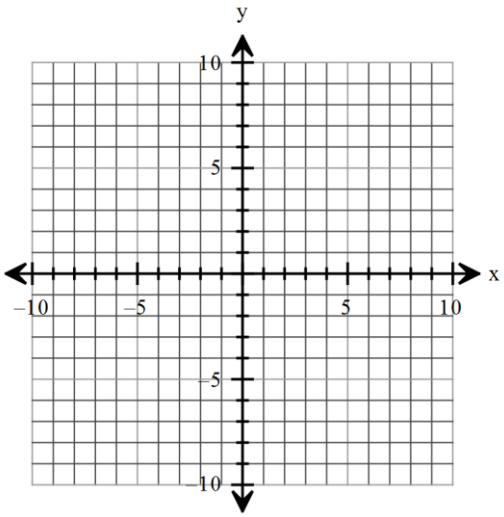
Domain: _____

Asymptotes:_____

Key points and transformations:

x	$f(x)$

x	$f(x)$



Range: _____

35. $f(x)=3\log_2(-x)$

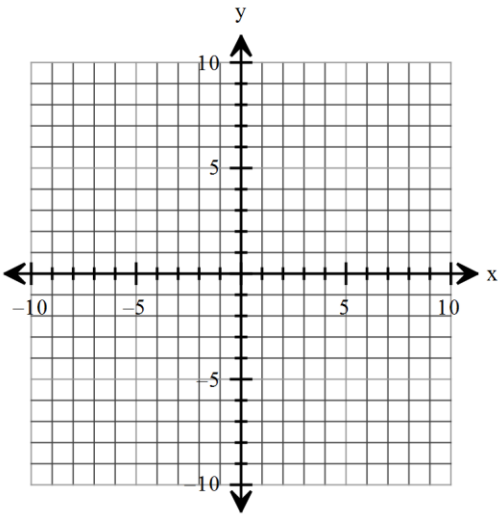
Domain: _____

Asymptotes:_____

Key points and transformations:

x	$f(x)$

x	$f(x)$



Range: _____

Solve each equation. Leave answer as exact solutions, no calculators. Then use your calculator and find the decimal approximation. Round to the nearest ten-thousandth. Write the restriction if needed. Show work!

36. $\log_2(2x+1)=3$

37. $\ln e^x = 5$

38. $\log_4 64 = x$

39. $\log_3 243 = 2x+1$

40. $e^{2x+5}=8$

41. $\log_2 8^x = -3$

42. $2\cdot 10^{2-x}=5$

43. $4\cdot e^{x+1}=5$

44. Chemists define the acidity or alkalinity of a substance according to the formula, $\text{pH} = -\log \text{H}^+$, where H^+ is the hydrogen ion concentration, measured in moles per liter. Solutions with a pH value of less than 7 are acidic. Solutions with a pH value of greater than 7 are basic. Solutions with a pH of 7 (such as pure water) are neutral.

a) Suppose you test apple juice and find that the hydrogen ion concentration is $\text{H}^+ = 0.0003$. Find the pH value and determine whether the juice is basic or acidic.

b) Suppose you test ammonia and find that the hydrogen ion concentration is $\text{H}^+ = 3.6 \times 10^{-12}$. Find the pH value and determine whether the juice is basic or acidic.

45. "Loudness" is measured in decibels. The formula for the loudness of a sound is given by $dB = 10 \log(\text{I} \div \text{I}_0)$ where I_0 is the intensity of "threshold sound", or sound that can barely be perceived. Other sounds are defined in terms of how many times more intense they are than threshold sound.

a) If a cat's purr is 316 times as intense as threshold sound or $\text{I} = 316\ \text{I}_0$, find the decibel rating.

b) Prolonged exposure to sounds above 85 decibels can cause hearing damage or loss. If a gunshot from a .22 rimfire rifle has an intensity, I, of about $(2.5 \times 10^{13})\text{I}_0$, should you wear ear protection when firing the rifle?

46. Earthquake intensity is measured by the Richter scale. The formula for the Richter rating of a given earthquake is given by $R = \log(\text{I} \div \text{I}_0)$ where I_0 is the threshold quake, or movement that can barely be detected, and the intensity, I, is given in terms of multiples of that threshold intensity. The seismograph says you have an event that had an intensity, I, of $989\ \text{I}_0$. A heavy truck can cause a microquake with a Richter rating of 3 or 3.5. A moderate quake has a Richter rating of 4 or more. Was the event more likely made from a heavy truck or a quake?