SM 3H

Date: **Section:**

Objective:

Question: What is the inverse of an exponential function? How do you solve for a variable that is in an exponent?

Find the inverse of $f(x) = 2^x$.

1. Replace $f(x)$ with y. _______________
2. Interchange x and y. _______________
3. Solve for y. _______________________________________
4. Replace y with $f^{-1}(x)$ _______________________________________

We need a new symbol to replace the words: "The exponent to which we raise 2 to get x":

$\log_2 x$ means "the exponent to which we raise 2 to get x."

Pronounced "the logarithm, base 2, of x" or "log, base 2, of x"

★LOGARITHMS ARE EXPONENTS!★

Logarithm: $\log_b a$ means ____

- b is called the ____
- a is called the ____

The ***logarithmic function of base a***, where $a > 0$ and $a \neq 1$ is denoted by $y = \log_a x$.

Formula for changing logarithmic functions to exponential functions:

Example: Change each exponential expression to an equivalent expression involving a logarithm.

a) $5^x = 625$ b) $x^3 = 64$ c) $3^2 = x$

Example: Change each logarithmic expression to an equivalent expression involving an exponent.

a) $\log_3 x = 5$ b) $\log_e 5 = x$ c) $\log_m 2 = n$

Evaluating Logarithms: It is helpful to replace "log" with the word "power".

- Instead of "$\log_2 8$," think "$\text{power}_2\, 8$." Ask yourself, what power of 2 equals 8?
  - The answer would be ________ because ________________________________

Example: Find the exact value of

a) $\log_3 9$ b) $\log_2 32$ c) $\log_6 1$ d) $\log_5 \dfrac{1}{125}$ e) $\log_7 \sqrt{7}$

Domain of a Logarithmic Function

The logarithmic function $y = \log_a x$ is ____, $y = a^x$.

Domain of the logarithmic function = ________________________________ $= (\_\_\_, \_\_\_)$

Range of the logarithmic function = ________________________________ $= (\_\_\_, \_\_\_)$

$y = \log_a x$ (defining equation: $x = a^y$)

Domain: $(0, \infty)$ Range: all real numbers

★ **Caution!** You can't take the log of zero or of a negative because it is impossible to get zero or a negative by raising a positive base to an exponent. **The argument of a logarithmic function must be greater than zero.**

Example: Find the domain of each logarithmic function by ____

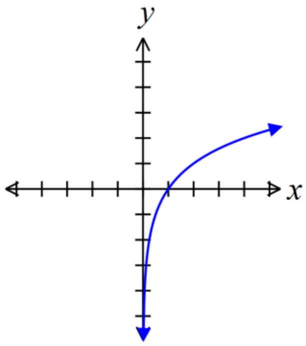
____

a) $f(x) = \log_2(x+3)$ b) $g(x) = \log_5\left(\dfrac{1+x}{1-x}\right)$ c) $h(x) = \log_{\frac{1}{2}}|x|$

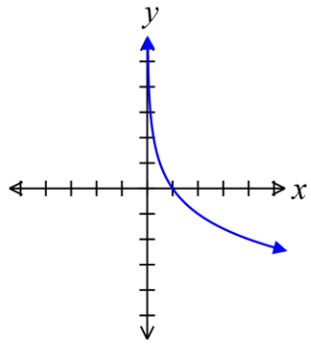
d) $f(x) = \log_3(5x-1)$ e) $f(x) = \log_7\left(\dfrac{1}{2x}\right)$

Graphs of Logarithmic Functions

$f(x) = \log_a x,\ a > 1$



$f(x) = \log_a x,\ 0 < a < 1$



Properties of the Logarithmic Function $f(x) = \log_a x$

1. The ____________ is the set of all positive real numbers; the __________ is the set of all real numbers.

2. The ________________ is 1. There is no __________________.

3. The ___________ or ____________ is a vertical asymptote of the graph.

4. The logarithmic function is ___________________ if $0 < a < 1$ and __________________ if $a > 1$. The function is one-to-one.
5. The graph of f contains the points _____________, _____________, and ________________.

6. The graph of f is smooth and continuous, with no corners, gaps, or cusps.

★ **Note:** It is often easier to graph a logarithmic function if you rewrite it as an exponential function first.

Graphing Logarithmic Functions using transformations:

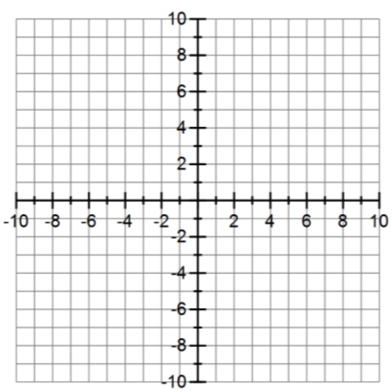
1. .

2. .

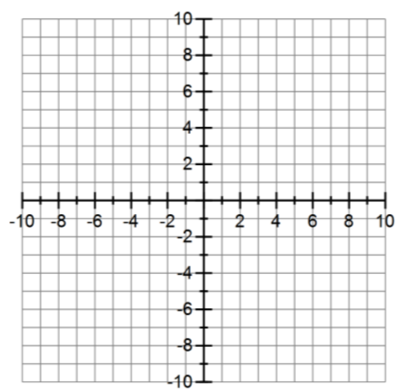
3. .

Examples: Write the transformations of each function. Graph each function using the 3 key points. Find domain, range, and vertical asymptote.

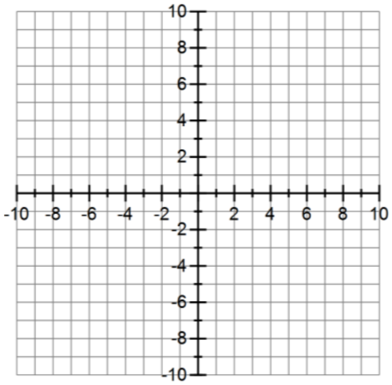
a) Graph $f(x) = \log_2(x-3)+2$



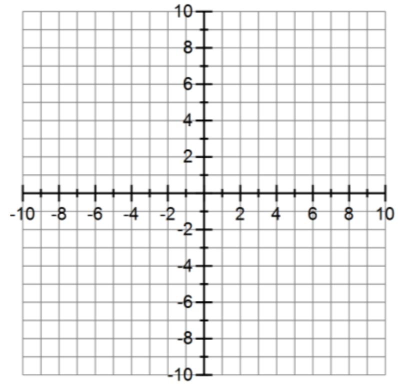
b) Graph $f(x) = -2\log_{10}(x)-3$



c) Graph $f(x) = \log_3(-x+1)$



d) Graph $f(x) = -\log_4(2x+1)+3$



Natural Logarithms: If the base of a logarithmic function is the number _____, then we have the natural logarithm function (abbreviated ln). That is, _________________ is inverse of ________________.

$f(x) = \ln a =$ ___________________

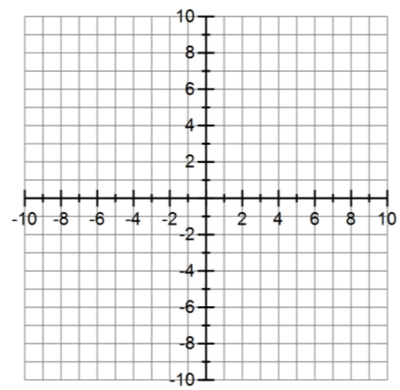
Example: $f(x) = -\ln(x+3)$

a) Find the domain of the logarithmic function.

b) Write the transformations

b) Graph $f(x)$ using the 3 key points

c) Find the range and vertical asymptote of f.



Common Logarithmic Function: If the base of a logarithmic function is the number ____, then we have the common logarithmic function. If the base a of the logarithmic function is not indicated, it is understood to be 10. That is, _______________ is inverse of _______________

$f(x) = \log a =$ ____________________

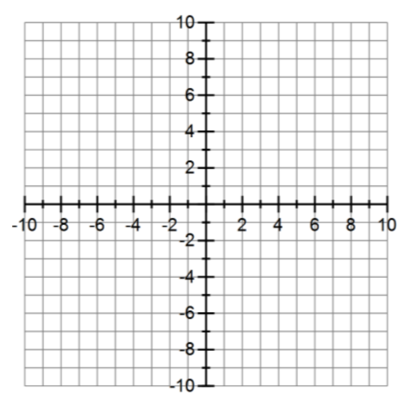
Example: $f(x) = 2\log(x-3)$

a) Find the domain of the logarithmic function.

b) Write the transformations

b) Graph $f(x)$ using the 3 key points

c) Find the range and vertical asymptote of f.



Use a calculator to evaluate each expression. Round your answer to three decimal places.

a. $\log 3.54$

b. $\log(-22)$

c. $\ln \frac{1}{4}$

d. $\frac{\ln 4}{6}$

e. $\frac{\log 2 + \log 6}{\ln 7 - \ln 5}$

Solving Logarithmic Equations

Many equations can be solved by rewriting logarithms as exponential functions or rewriting exponential functions as logarithms.

★ When solving logarithmic equations, remember that in the expression $\log_a M$, a and M must be positive and $a \neq 1$. Be sure to check each solution in the original equation and discard any that are extraneous.

Examples: Solve the logarithmic equations

a) $\log_3(3x-2)=2$

b) $\log_x\left(\frac{1}{8}\right)=3$

c) $10^{2x-7}=3$

d) $e^{3x-2}=7$

e) $\log_2(x^2+2x)=3$

f) $8e^{x+2}=3$