SM 3H

Date: 12/15/23 **Section:** 5.2

Objective: I can graph and rewrite logarithmic functions.

Question: What is the inverse of an exponential function? How do you solve for a variable that is in an exponent?

Find the inverse of $f(x) = 2^x$.

1. Replace $f(x)$ with y. ______________
2. Interchange x and y. ______________
3. Solve for y. ______________________________
4. Replace y with $f^{-1}(x)$ ______________________________

We need a new symbol to replace the words: "The exponent to which we raise 2 to get x":

$\log_2 x$ means "the exponent to which we raise 2 to get x."

Pronounced "the logarithm, base 2, of x" or "log, base 2, of x"

★LOGARITHMS ARE EXPONENTS!★

Logarithm: **$\log_b a$** means log base b of a, what is exponent of b to get a?

- b is called the base
- a is called the argument

The ***logarithmic function of base a***, where $a > 0$ and $a \neq 1$ is denoted by $y = \log_a x$.

Formula for changing logarithmic functions to exponential functions:

$\log_a x = y$ $a^x = y$ $a^x = y$ $\log_a x = y$

$x = a^y$ $x = \log_a y$ $\log_a y = x$ $a^y = x$

Example: Change each exponential expression to an equivalent expression involving a logarithm.

a) $5^x = 625$ inverse

$x = \log_5 625$

b) $x^3 = 64$

$\log_x 64 = 3$

c) $3^2 = x$

$\log_3 x = 2$

Example: Change each logarithmic expression to an equivalent expression involving an exponent.

a) $\log_3 x = 5$

$x = 3^5$

b) $\log_e 5 = x$

$e^x = 5$

c) $\log_m 2 = n$

$m^n = 2$

Evaluating Logarithms: It is helpful to replace "log" with the word "power".

- Instead of "$\log_2 8$," think "$\text{power}_2 8$." Ask yourself, what power of 2 equals 8?
  - The answer would be ________ because ______________________________

Example: Find the exact value of

a) $\log_3 9$ b) $\log_2 32$ c) $\log_6 1$ d) $\log_5 \dfrac{1}{125}$ e) $\log_7 \sqrt{7}$

a) 3 to the what is 9?

2

b) 5

c) 0

d) -3

e) $\log_7 7^{\frac{1}{2}}$

$\frac{1}{2}$

Domain of a Logarithmic Function

The logarithmic function $y = \log_a x$ is ______________________________, $y = a^x$.

Domain of the logarithmic function = $x > 0$ = $(0, \infty)$

Range of the logarithmic function = ______________________________ = (___, ___)

$y = \log_a x$ (defining equation: $x = a^y$)

Domain: $(0, \infty)$ Range: all real numbers

★ **Caution!** You can't take the log of zero or of a negative because it is impossible to get zero or a negative by raising a positive base to an exponent. **The argument of a logarithmic function must be greater than zero.**

Example: Find the domain of each logarithmic function by set argument > 0 & solve

a) $f(x) = \log_2 (x+3)$

$x + 3 > 0$

$x > -3$

$(-3, \infty)$

b) $g(x) = \log_5 \left(\dfrac{1+x}{1-x} \right)$ $x \neq 1$

$\dfrac{1+x}{1-x} > 0$

$1 + x > 0$

$x > -1$

$(-1, 1) \cup (1, \infty)$

c) $h(x) = \log_{\frac{1}{2}} |x|$ $x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

d) $f(x) = \log_3 (5x - 1)$

$5x - 1 > 0$

$x > \frac{1}{5}$

$(\frac{1}{3}, \infty)$

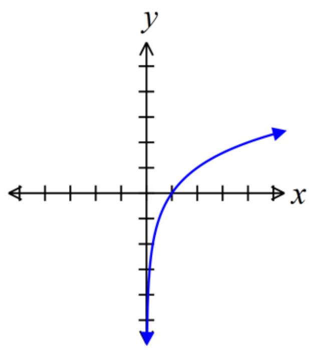
e) $f(x) = \log_7 \left(\dfrac{1}{2x} \right)$

$\dfrac{1}{2x} > 0$

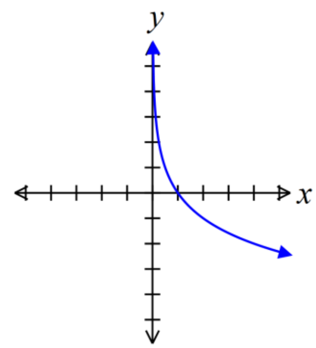
$(0, \infty)$

Graphs of Logarithmic Functions

$f(x) = \log_a x,\ a > 1$



$f(x) = \log_a x,\ 0 < a < 1$



Properties of the Logarithmic Function $f(x) = \log_a x$

1. The ____________ is the set of all positive real numbers; the __________ is the set of all real numbers.

2. The ________________ is 1. There is no __________________.

3. The ___________ or ____________ is a vertical asymptote of the graph.

4. The logarithmic function is ___________________ if $0 < a < 1$ and __________________ if $a > 1$. The function is one-to-one.
5. The graph of f contains the points _____________, _____________, and ________________.

6. The graph of f is smooth and continuous, with no corners, gaps, or cusps.

★ **Note:** It is often easier to graph a logarithmic function if you rewrite it as an exponential function first.

Graphing Logarithmic Functions using transformations:

1. .

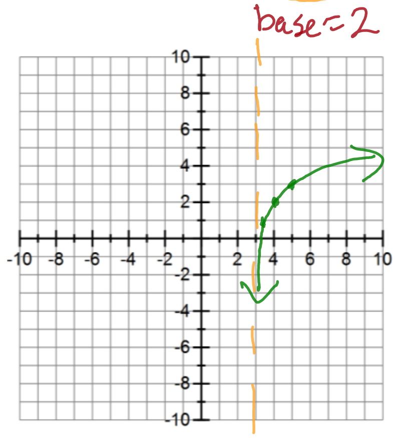
2. .

3. .

Examples: Write the transformations of each function. Graph each function using the 3 key points. Find domain, range, and vertical asymptote.

a) Graph $f(x) = \log_2(x-3) + 2$ translate right 3 up 2

base=2



x	y
$\frac{1}{2}$	-1
1	0
2	1

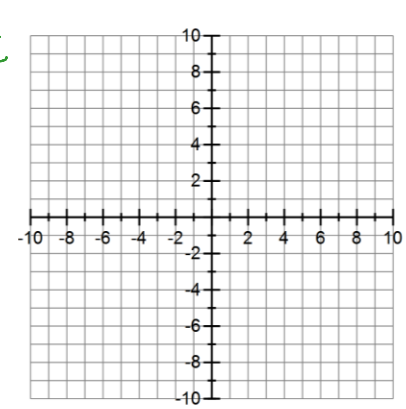
→

x+3	y+2
$3\frac{1}{2}$	1
4	2
5	3

D=(3,∞)

X=3

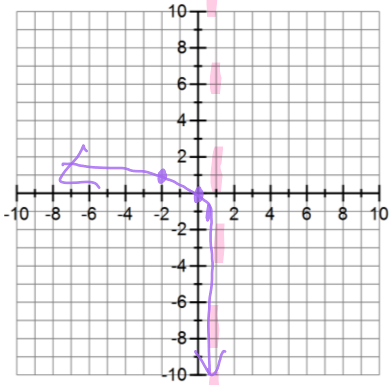
b) Graph $f(x) = -2\log_{10}(x) - 3$



$\frac{2x+1}{2}\frac{}{2} = 2\left(x+\frac{1}{2}\right)$

c) Graph $f(x) = \log_3(-x+1) = \log_3(-(x-1))$

- reflect over y-axis
- translate right 1



x	y
$\frac{1}{3}$	-1
1	0
3	1

$\to$

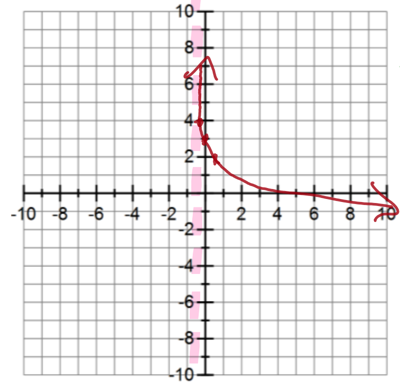
$-x+1$	y
$\frac{2}{3}$	-1
0	0
-2	1

$-x+1>0$

$-x>-1$ $\quad (-\infty, 1)$

$x<1$

d) Graph $f(x) = -\log_4(2x+1)+3 = -\log_4\left(2\left(x+\frac{1}{2}\right)\right)+3$



x	y
$\frac{1}{4}$	-1
1	0
4	1

$\to$

$\frac{x}{2}-\frac{1}{2}$	$-y+3$
$-\frac{3}{8}$	4
0	3
$1\frac{1}{2}$	2

$2x+1>0$

$x>-\frac{1}{2}$

Natural Logarithms: If the base of a logarithmic function is the number $\underline{e}$, then we have the natural logarithm function (abbreviated ln). That is, $\underline{\log_e x = y}$ is inverse of $\underline{e^y = x}$.

$f(x) = \ln a = \underline{\log_e x}$

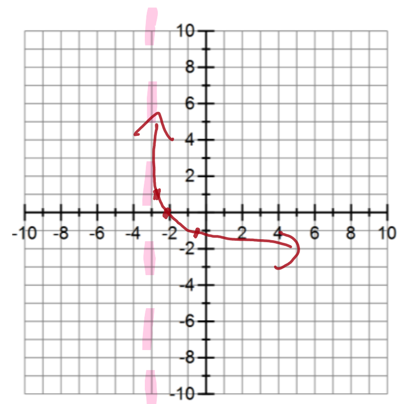
Example: $f(x) = -\ln(x+3)$

a) Find the domain of the logarithmic function. $(-3, \infty)$

$x+3>0 \quad x>-3$

b) Write the transformations reflect over y-axis, translate left 3

b) Graph $f(x)$ using the 3 key points



c) Find the range and vertical asymptote of f. $(-\infty, \infty)$

$x=-3$

x	y
$\frac{1}{e}$	-1
1	0
e	1

$\to$

$x-3$	$-y$
$-2.6 \approx \frac{1}{e}-3$	1
-2	0
$-.3 \approx e-3$	1

Common Logarithmic Function: If the base of a logarithmic function is the number $\underline{10}$, then we have the common logarithmic function. If the base a of the logarithmic function is not indicated, it is understood to be 10. That is, $\underline{\log_{10} x = y}$ is inverse of $\underline{10^y = x}$

$f(x) = \log a = \underline{\log_{10} a}$

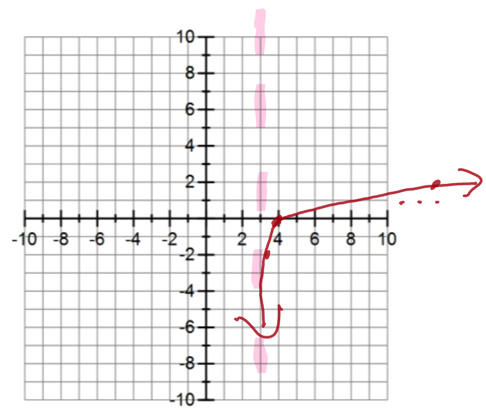
Example: $f(x) = 2\log(x-3)$

a) Find the domain of the logarithmic function. $(3, \infty)$

$x-3>0 \quad x>3$

b) Write the transformations vert. stretch of 2, translate right 3

b) Graph $f(x)$ using the 3 key points



c) Find the range and vertical asymptote of f. $(-\infty, \infty)$

$x=3$

x	y
$\frac{1}{10}$	-1
1	0
10	1

$\to$

$x+3$	$2y$
$3\frac{1}{10}$	-2
4	0
13	2

Use a calculator to evaluate each expression. Round your answer to three decimal places.

a. $\log 3.54 \approx 0.549$

b. $\log(-22) \approx$ no sol.

c. $\ln \frac{1}{4}$

d. $\frac{\ln(4)}{6} \approx .231$

e. $\frac{(\log(2) + \log(6))}{(\ln(7) - \ln(5))} \approx 3.207$

Solving Logarithmic Equations

Many equations can be solved by rewriting logarithms as exponential functions or rewriting exponential functions as logarithms.

★ When solving logarithmic equations, remember that in the expression $\log_a M$, a and M must be positive and $a \neq 1$. Be sure to check each solution in the original equation and discard any that are extraneous.

Examples: Solve the logarithmic equations

a) $\log_3(3x-2) = 2$ $\quad x > \frac{2}{3}$

3 $\quad 3$

$3^2 = 3x - 2$

$11 = 3x$

$x = \frac{11}{3}$

b) $\log_x\left(\frac{1}{8}\right) = 3$

c) $10^{2x-7} = 3$

d) $\cancel{\ln} e^{3x-2} = 7$

$\ln$ $\quad \ln$

$3x - 2 = \ln 7$

$3x = \ln(7) + 2$

$x = \frac{\ln(7)+2}{3}$ exact ans.

$x \approx 2.613$

e) $\log_2(x^2 + 2x) = 3$

f) $\frac{8e^{x+2}}{8} = \frac{3}{8}$

$e^{x+2} = \frac{3}{8}$

$\ln \frac{3}{8} = x + 2$

$\ln\left(\frac{3}{8}\right) - 2 = x$ exact ans