SM 3H

Date: 12/19/23 **Section:** 5.3

Objective: I can use log properties.

******Memorize the following:**

***Definition of Logarithm*:** $y = \log_a x \Leftrightarrow$ $a^y = x$

***Properties of Logarithms*:** For any positive numbers M, N, and a, where $a \neq 1$ and r is any real number:

$\log_a 1 =$ 0 ($a^0 = 1$) $\log_a a^1 =$ 1 ($a^1 = a$) $a^{\log_a M} =$ M $\log_a a^r =$ r ($a^? = a^r$)

$\log_a(MN) =$ $\log_a M + \log_a N$

$\log_a M^r =$ $r \log_a M$ $(x^m)^n$

$\log_a\left(\frac{M}{N}\right) =$ $\log_a M - \log_a N$

$\log_a M = \log_a N \Leftrightarrow$ $M = N$

***Change of Base Formula*:** used to type on calc.

$\log_a M = \dfrac{\log M}{\log a}$ $\log_a M = \dfrac{\ln M}{\ln a}$ $\log_a M = \dfrac{\log_b M}{\log_b a}$

Examples: Find the exact value of each expression. (Do not use a calculator).

a) $\log_{0.6} 0.6^{-3.2}$ −3.2

b) $5^{\log_5 3}$ 3

c) $\log_7 7^{-1}$ −1

d) $e^{\ln 12}$ 12

Examples: Use the change of base formula to evaluate each logarithm. Round to the nearest ten-thousandths.

a) $\log_6 9$ $\dfrac{\log 9}{\log 6} \approx 1.2263$ $\dfrac{\ln 9}{\ln 6} \approx 1.2263$

b) $\log_{\sqrt{2}} 7$ $\dfrac{\log 7}{\log \sqrt{2}} \approx 5.6147$

c) $\log_\pi \sqrt{3}$

d) $\log_3 5$

Examples: Use properties of logarithms to find the exact value of each expression. (Do not use a calculator).

a) $\log_4 36 - \log_4 9$ $\log_4 \frac{36}{9}$ $\log_4 4$ 1

b) $5^{\log_5 6 + \log_5 7}$ $5^{\log_5 42}$ 42

c) $e^{\log_{e^2} 9}$ $e^{\log_{e^2} 3^2}$ 3

d) $\log_2 6 \cdot \log_6 16$ $\dfrac{\log 6}{\log 2} \cdot \dfrac{\log 16}{\log 6}$ $\dfrac{\log 16}{\log 2}$ $\log_2 2^4 \leftarrow \log_2 16 = 4$ $2^? = 16$

Expand each log.

Examples: Write each expression as a sum/difference of logarithms. Express powers as factors.
Another way to write the directions: Expand each logarithm.

a) $\log_7(x^5)$ $= 5\log_7 x$

b) $\ln(xe^x)$

$\ln x + \ln e^x$

$\ln(x) + x$

c) $\log_2\left(\frac{a}{b^2c}\right)$, $a>0,\ b>0,\ c>0$

$\log_2 a - \log_2 b^2c$

$\log_2 a - (\log_2 b^2 + \log_2 c)$

$\log_2 a - 2\log_2 b - \log_2 c$

d) $\ln\left[\frac{(x-4)^2}{x^2-1}\right]^{2/3}$; $x>4$

$\frac{2}{3}\ln\left(\frac{(x-4)^2}{x^2-1}\right)$

$\frac{2}{3}\left(\ln(x-4)^2 - \ln(x^2-1)\right)$

$\frac{2}{3}\left(2\ln(x-4) - \ln(x^2-1)\right)$

OR $\frac{4}{3}\ln(x-4) - \frac{2}{3}\ln(x^2-1)$

Examples: Write each expression as a single logarithm.
Another way to write the directions: Condense each logarithm.

a) $3\log_5 u + 4\log_5 v$

$\log_5 u^3 + \log_5 v^4$

$\log_5 u^3v^4$

b) $\log_4(x^2-1) - 5\log_4(x+1)$

$\log_4(x^2-1) - \log_4(x+1)^5$

$\log_4\left(\frac{x^2-1}{(x+1)^5}\right)$

$\frac{(x-1)(x+1)}{(x+1)^5}$

$\log_4\left(\frac{x-1}{(x+1)^4}\right)$

c) $\log\left(\frac{x^2-2x-3}{x^2-4}\right) - \log\left(\frac{x^2+7x+6}{x+2}\right)$

$\log\left(\frac{\frac{x^2-2x-3}{x^2-4}}{\frac{x^2+7x+6}{x+2}}\right)$

$\frac{(x-3)(x+1)}{(x+2)(x-2)} \cdot \frac{x+2}{(x+6)(x+1)}$

$\log\left(\frac{x-3}{(x-2)(x+6)}\right)$

d) $21\log_3 \sqrt[3]{x} + \log_3(9x^2) - \log_3 9$

$\log_3 \sqrt[3]{x}^{21} + \log_3(9x^2) - \log_3 9$

$\log_3 x^{\frac{21}{3}} + \log_3(9x^2) - \log_3 9$

$\log_3 x^7 + \log_3(9x^2) - \log_3 9$

$\log_3 9x^9 - \log_3 9$

$\log_3 x^9$

e) $\frac{1}{3}\log(x^3+1) + \frac{1}{2}\log(x^2+1)$

$\log(x^3+1)^{\frac{1}{3}}(x^2+1)^{\frac{1}{2}}$

OR $\log \sqrt[3]{x^3+1} \cdot \sqrt{x^2+1}$