

Date: 1/4/24

Section: 5.4

Objective: I can solve logarithmic and exponential equations.

Solve a logarithmic or exponential function—the object is to try to get the log or exponent by itself so you can do the inverse of it

There is not one specific set of rules for you to follow. I can only give you hints.

HINTS:

- Try to get the variable by itself ☆
- Isolate the logarithm or the exponent by getting rid of the add or subtract, then the multiply or divide
- Use the logarithm properties to make 2 logs into 1 log
- Once the log or exponent is by itself, do the inverse ("swirl", make the L or e)
- Remember that if a log has no variables, it is a constant and can sometimes be changed to a number. EXAMPLE: $\log_5 25$ has no variables so change it to the number 2
 - Only do this if the number is a nice number, not a nasty decimal
- Logs CANNOT be negative, so check your answers against the domain since some are extraneous
- If there is one log with the same base on each side of the equal sign, you can eliminate the log

EXAMPLES: Solve the following equations. Write the restrictions and check for extraneous answers.

1. $\log_7 6 \ominus \log_7(x-2) = 2$ $x > 2$
(2, ∞)

$$\log_7 \frac{6}{x-2} = 2$$

$$\frac{6}{x-2} = 49$$

$$6 = 49x - 98$$

$$104 = 49x$$

$$x = \frac{104}{49}$$

2. $\log_8(x+2) \ominus \log_8 x = \log_8 67$ $x > -2$
 $x > 0$

$$\log_8 \frac{x+2}{x} = \log_8 67$$
 OR $\log_8 \frac{x+2}{x} - \log_8 67 = 0$

$$\frac{x+2}{x} = 67$$

$$x+2 = 67x$$

$$2 = 66x$$

$$x = \frac{1}{33}$$

3. $4^{x+5} - 3 = 9$ $(-\infty, \infty)$

$$\log_4 4^{x+5} = \log_4 12$$

$$x+5 = \log_4 12$$

$$x = \log_4 12 - 5$$

$$x = \frac{\log 12}{\log 4} - 5$$

$$x \approx -3.2075$$

4. $15 \left(\frac{1}{3}\right)^{\frac{x}{4}} = 90$

$$\left(\frac{1}{3}\right)^{\frac{x}{4}} = 6$$

$$\left(\frac{1}{3}\right)^6 = \frac{x}{4}$$

$$4 \left(\frac{1}{3}\right)^6 = x$$

$$x = \frac{4}{729}$$

5. $\textcircled{3} \log_2(x-1) \oplus \log_2 4 = 5$ $x > 1$

$$\log_2(x-1)^3 + \log_2 4 = 5$$
 OR $3 \log_2(x-1) + 2 = 5$

$$2 \log_2 4(x-1)^3 = 5$$

$$3 \log_2(x-1) = 3$$

$$\log_2(x-1) = 1$$

$$2 = x-1$$

$$x = 3$$

$$4(x-1)^3 = 32$$

$$(x-1)^3 = 8$$

$$x-1 = 2$$

$$x = 3$$

6. $\log_6(x+4) \oplus \log_6(x+3) = 1$ $x > -3$

$$\log_6(x^2 + 7x + 12) = 1$$

$$6^1 = x^2 + 7x + 12$$

$$x^2 + 7x + 6 = 0$$

$$(x+1)(x+6) = 0$$

$$x = -1, -6$$