

Date: 1/10/24

Section: 5.5

Objective: ① I can find The inverse.  
 ② I can solve using "U" substitution.

Steps for finding the inverse:

1. Flip-flop the  $x$  and  $y$
2. Solve for  $y$

**EXAMPLES:** Find the inverse.

$$1. f(x) = \log_8(2x+1) - 6$$

$$x = \log_8(2y+1) - 6$$

$$f^{x+6} = \log_8(2y+1)$$

$$8^{x+6} = 2y+1$$

$$8^{x+6} - 1 = 2y$$

$$\frac{8^{x+6} - 1}{2} = f^{-1}(x)$$

$$2. f(x) = 2 \cdot 3^{x+1} - 4$$

$$3. f(x) = \frac{e^{\frac{5x-1}{2}} + 8}{3}$$

$$x = \frac{e^{\frac{5y-1}{2}} + 8}{3}$$

$$3x = e^{\frac{5y-1}{2}} + 8$$

$$\ln(3x-8) = \frac{5y-1}{2}$$

$$\ln(3x-8) = \frac{5y-1}{2}$$

$$2 \ln(3x-8) = 5y-1$$

$$2 \ln(3x-8) + 1 = 5y$$

$$\frac{2 \ln(3x-8) + 1}{5} = f^{-1}(x)$$

$$\frac{2}{5} \ln(3x-8) + \frac{1}{5}$$

Steps for "U" substitution:

1. Set  $u$  equal to the middle term's variable
2. Square step #1 to find  $u^2$
3. Replace  $u$  and  $u^2$  in the original equation for the variables
4. Factor
5. Solve for  $u$
6. Substitute  $x$  back in and solve for  $x$
7. Check for extraneous solutions

**EXAMPLES:** Solve for the variable.

$$1. e^{6x} + 4e^{3x} - 32 = 0 \quad u = e^{3x}$$

$$u^2 + 4u - 32 = 0$$

$$(u+8)(u-4) = 0$$

$$u = -8 \quad u = 4$$

~~$$e^{3x} = 8 \quad \ln e^{3x} = \ln 4$$~~

$$3x = \ln 4$$

3

$$x \approx .4621$$

$$2. 2 \cdot 6^{4x} - 6^{2x} - 6 = 0 \quad u = 6^{2x}$$

$$2u^2 - u - 6 = 0$$

$$(2u+3)(u-2) = 0$$

$$u = -\frac{3}{2} \quad u = 2$$

~~$$6^{2x} = -\frac{3}{2} \quad \log_6 6^{2x} = \log_6 2$$~~

$$x = \frac{\log_2 2}{\log_6 2} \div 2$$

$$\frac{2x = \log_6 2}{2}$$

$$x \approx .1934$$