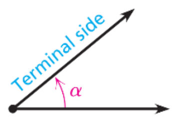
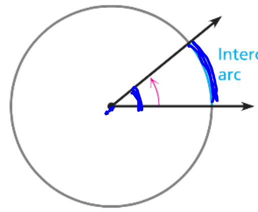
SM 3H

Date: 1/24/24 **Section:** 6.1
Objective: I can find angles measures + arc length.

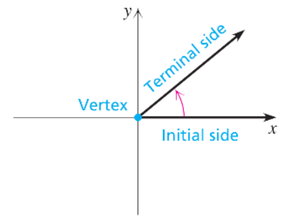
An angle can be formed by rotating one ray away from a fixed ray indicated by an arrow. The fixed ray is the **initial side** and the rotated ray is the terminal side. An angle whose vertex is the center of a circle is a central angle, and the arc of the circle through which the terminal side moves is the intercepted arc. An angle in standard position is located in a rectangular coordinate system with the vertex at the origin and the initial side on the positive x-axis.



Angle α



Central angle



Angle in standard position

Greek letters used for the variable of an unknown angle:
$\alpha =$ alpha $\beta =$ beta $\gamma =$ gamma $\theta =$ theta

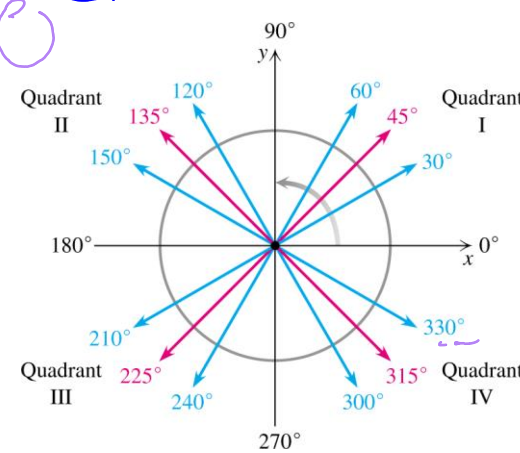
Degree Measure of Angles

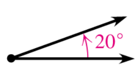
The measure, $m(\alpha)$, of an angle α is the amount of rotation from the initial side to the terminal side, and is found by using any circle centered at the vertex. An angle that forms a complete circle arc is 360°.

The **degree measure of an angle** is the number of degrees in the intercepted arc of a circle centered at the vertex.

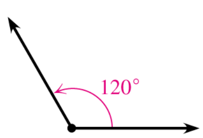
Counterclockwise rotation—positive angles
Clockwise rotation—negative angles

An angle in standard position is said to lie in the quadrant where its terminal side lies.

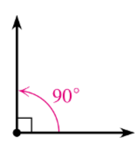




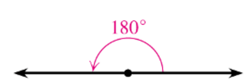
Acute angle



Obtuse angle



Right angle

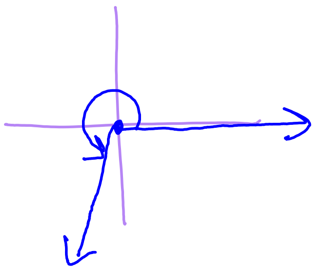


Straight angle

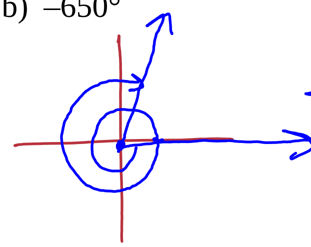
Acute angle—$x < 90°$
Obtuse angle— $90° < x < 180°$
Right angle— $x = 90°$
Straight angle— $x = 180°$
Quadrantal Angle— axis angles

Example: Draw each angle in standard position, then name the quadrant in which the terminal side lies.

a) 255°

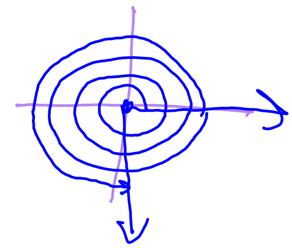


b) –650° – coterminal angle



$$\begin{array}{r} 650 \\ -\ 360° \\ \hline 290° \end{array}$$

c) 1360°



How do you measure the line? ______________________________ in meters dm cm units

https://commons.wikimedia.org/wiki/File:Circle_radians.gif#/media/File:Circle_radians.gif

Unit circle: A circle of radius one that is centered at the origin.

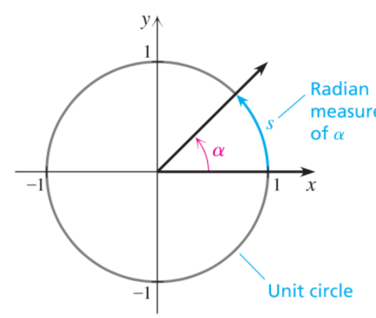
What is the circumference of the unit circle? (Circumference of a circle: $C = \pi d = 2\pi r$)

What is the arc length intercepted by a 180° angle (1/2 of the circle)?

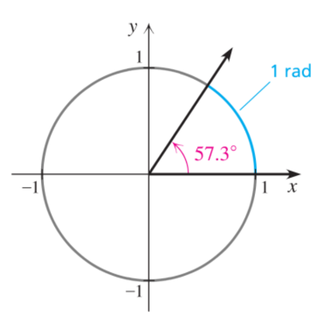
What is the arc length intercepted by a 120° angle (1/3 of the circle)?

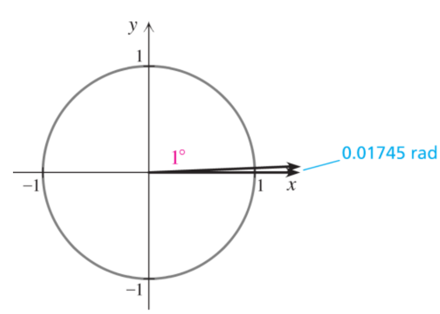
What is the arc length intercepted by a 30° angle? An angle of 225°? An angle of 210°?

You have just calculated the radian measure of each of these angles.
The __radian__ __measure__ of the angle α in standard position is the directed length of the intercepted arc on the unit circle. ________________ ______________ means that the radian measure is positive if the rotation of the terminal side is counterclockwise and negative if the rotation is clockwise.



One radian: The angle that intercepts an arc with length equal to the radius of a circle. (On the unit circle, one radian is the angle that intercepts an arc of length one.)





Since the radius of the unit circle is the real number 1, without any dimension (ft., meters, etc.), the arc length, and hence the radian measure of an angle, is also a real number without any dimension. Thus, one radian (1 rad) is the real number 1.

Converting Between Radians and Degrees:

Since there are 2π radians in a circle (the circumference of the unit circle is 2π) and 360° in a circle,

$$2\pi \text{ radians} = 360°, \text{ or } \pi \text{ radians} = 180°.$$

Degrees→Radians: multiply by $\frac{\pi}{180°}$ **Radians→Degrees:** multiply by $\frac{180°}{\pi}$

Examples:

Convert the degree measures to radians:

a) $210°\left(\frac{\pi}{180°}\right)$

$\frac{7\pi}{6}$

b) $-27.2°\left(\frac{\pi}{180°}\right)$

$-\frac{34\pi}{225}$ exact $-.4747$ round

Convert the radian measures to degrees:

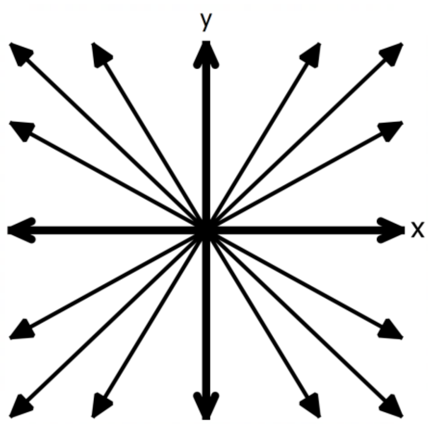
a) $\frac{5\cancel{\pi}}{3}\left(\frac{180°}{\cancel{\pi}}\right)$

$300°$

b) 16.7 radians $\left(\frac{180°}{\pi}\right)$

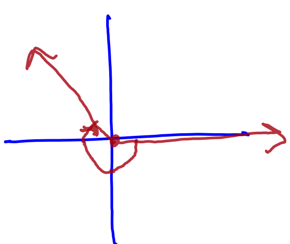
$956.8395°$

Rectangular Coordinate System

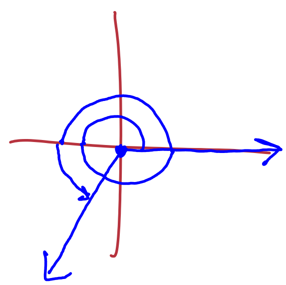


Examples: Draw each angle in standard position and determine the quadrant in which each angle lies.

a) $-\frac{5\pi}{4}$



b) $\frac{10\pi}{3}$



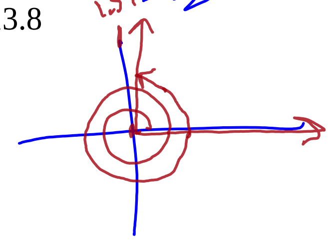
c) 13.8

$1.57 = \frac{\pi}{2}$

$13.80 - 12.56 \sim 4\pi$

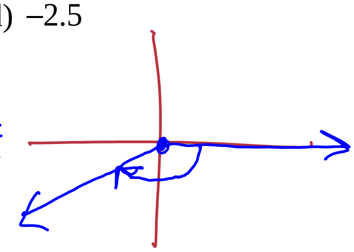
1.24

$6.28 = 2\pi$



d) -2.5

3.14

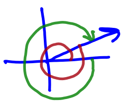


The terminal side of an angle may be rotated in either a positive or negative direction to get to its final position. It can also be rotated for more than one revolution in either direction. Two angles with the same terminal side are called **coterminal angles.**

Coterminal Angles—Angles α and β are coterminal if and only if there is an integer k such that $m(\beta) = m(\alpha) + 360°k$ or $m(\beta) = m(\alpha) + 2\pi k$. To find coterminal angles in degrees, add and subtract multiples of 360°. To find coterminal angles in radians, add or subtract multiples of 2π. Make sure to find a common denominator.

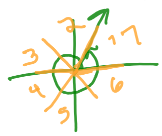
Examples: Find a positive angle and a negative angle that are coterminal with each angle: $2\pi \approx 6.28$

a) 23° $23° + 360° =$ $383°$ $23° - 360°$ $-337°$ $23 \pm 360°$



b) –146°

c) $\frac{7\pi}{3}$ $\pm \frac{6\pi}{3} = \frac{13\pi}{3}$ $\frac{\pi}{3} - \frac{6\pi}{3}$ $-\frac{5\pi}{3}$



d) $-\frac{\pi}{4}$

e) 1.4 ± 6.28 7.68, −4.88

Examples: Determine whether the angles in each pair are coterminal: prove

a) –128° and 592°

$-128° + 360° = 232°$

$232° + 360° = 592°$

yes

b) 8° and –368°

c) $\frac{3\pi}{2}$ and $\frac{9\pi}{2}$ $+2\pi$

$\frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$

no

What is the circumference of the unit circle? (Circumference of a circle: $C = \pi d = 2\pi r$)

$r = 1$ $C = 2\pi$

What is the arc length intercepted by a 180° angle (1/2 of the circle)?

$\frac{2\pi}{2} = \pi$

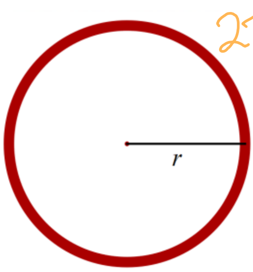
What is the arc length intercepted by a 330° angle?

$330\left(\frac{1}{180°}\right) = \frac{11\pi}{6}$

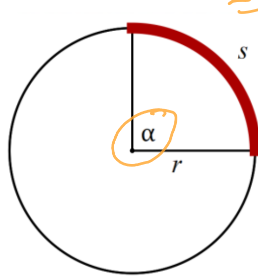
What if the radius isn't 1? How do we find the arc length of the intercepted arc?

Arc Length

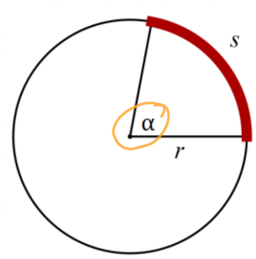
Often, we want to find the arc length, s, on a circle of radius, r, intercepted by an angle, α. We can do this by determining what fraction of the circle we are looking at, then multiplying by the circumference of the circle.



$2\pi r$



$\frac{2\pi r}{4}$



$2\pi r\left(\frac{\alpha}{360°}\right)$

$2\pi r\left(\frac{\alpha}{2\pi}\right)$

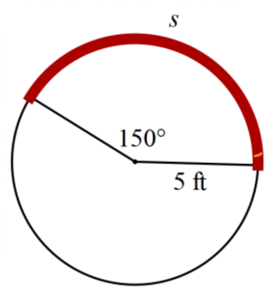
Degrees: $2\pi r\left(\frac{\alpha}{360^\circ}\right)$

Radians: $2\pi r\left(\frac{\alpha}{2\pi}\right)$ or $r\alpha$ **This formula only works if α is in radians!**

Examples:

Use the arc length formula and the given information to find the arc length. If you are given radians, write the answer in terms of π and use the π button to round answers to the nearest tenth. If you are given degrees, round the answer to the nearest tenth of a degree. Show work using the formula. Remember, leave work and answer in the units it begins with.

$a.\ r = 5\text{ ft.}, \theta = 150^\circ$



$2\pi(5)\left(\frac{150^\circ}{360^\circ}\right)$

exact: don't use use π

$\frac{25\pi}{6}$ ft

round: use π button ≈ 13.1 ft

$b.\ r = 2.5\text{ km}, \theta = \frac{5\pi}{3}$

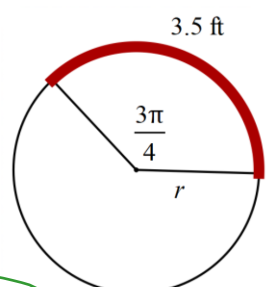
$\cancel{2\pi}(2.5)\left(\frac{\frac{5\pi}{3}}{\cancel{2\pi}}\right)$

$\frac{25\pi}{6}$ km.

13.1 km

Use the arc length formula and the given information to find the indicated measure. Round answers to the nearest tenth if necessary. Show work using the formula. Remember, leave work and answer in the units it begins with.

$c.\ \boxed{s = 3.5}\ ft., \theta = \frac{3\pi}{4}\ rad;$ find r



$\frac{3.5 = \cancel{2\pi} r\left(\frac{\frac{3\pi}{4}}{\cancel{2\pi}}\right)}{\frac{3\pi}{4}}$

$r \approx 1.5$ ft

$d.\ \ r = 3\text{ in.}, s = 20.5;$ find θ in (leave answer in degrees)

$\frac{360}{6\pi}\left(20.5 = 2\pi\, 3\left(\frac{\theta}{360^\circ}\right)\right)$

391.5°