SM 3H

Date: 1/26/24 **Section:** 6.2

Objective: I can use sector area and arc length.
I can find linear and angular velocity.

What is the area of a unit circle? (Area of a circle: $A = \pi r^2$)

What is the area of the section of the circle when the central angle is 180° (1/2 of the circle)?

What is the area of the section of the circle when the central angle is 225° angle?

What if the radius isn't 1? How do you find the area of a section of the circle?

Area of a Sector of a Circle

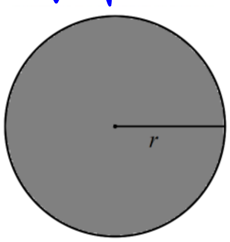
Finding the area (A) of a sector of a circle of radius r with central angle α, is similar to finding the arc length:

Determine what fraction of the circle the sector makes up, then multiply by the area of the circle.

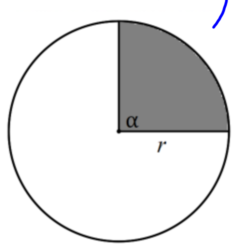
$2\pi r$ = linear circumference

πr^2 = 2-D

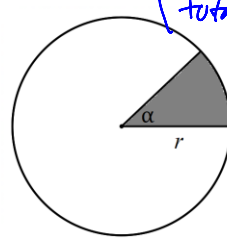
πr^2



$\pi r^2\left(\frac{1}{4}\right)$



$\pi r^2\left(\frac{\alpha}{\text{total}}\right)$



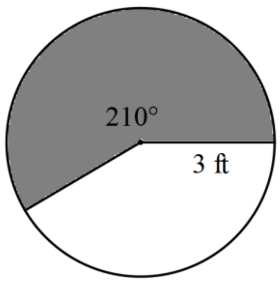
Degrees: $\pi r^2\left(\frac{\alpha}{360°}\right)$

Radians: $\pi r^2\left(\frac{\alpha}{2\pi}\right)$ or $\frac{r^2\alpha}{2}$ **This formula only works if α is in radians!**

Examples:

Use the sector area formula and the given information to find the sector area. If you are given radians, write the answer in terms of π and use the π button to round answers to the nearest tenth. If you are given degrees, round the answer to the nearest tenth of a degree. Show work using the formula. Remember, leave work and answer in the units it begins with.

$a.\ r = 3 \text{ ft.},\ \alpha = 210°$



$\pi 3^2\left(\frac{210°}{360°}\right)$

$\frac{21\pi}{4}$ ft^2 exact

16.5 ft^2 round

$b.\ r = 10 \text{ cm},\ \alpha = \frac{11\pi}{6}$

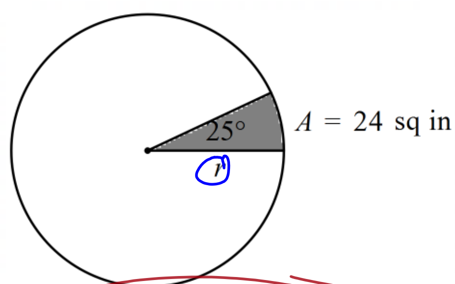
$\pi 10^2\left(\frac{\frac{11\pi}{6}}{2\pi}\right)$

$\frac{275}{3}\pi$ cm^2

288.0 cm^2

Use the sector area formula and the given information to find the indicated measure. Round answers to the nearest tenth if necessary. Show work using the formula. Remember, leave work and answer in the units it begins with.

$c.\ \alpha = 25°,\ A = 24 \text{ sq. in; find } r$



$\frac{360°}{25\pi}\left(24 = \pi r^2\left(\frac{25°}{360°}\right)\right)$

$\sqrt{r^2} = \sqrt{110.0}$

$r \approx 10.5$ in

$d.\ r = 3 \text{ ft.},\ A = 36 \text{ sq. ft.; find } \alpha$ (leave answer in radians)

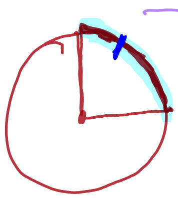
$\frac{2}{9}\left(36 = \pi 3^2\left(\frac{\alpha}{2\pi}\right)\right)$

$8 = \alpha$

Now let's apply arc length and sector area in real-world applications.

<u>EXAMPLE:</u>

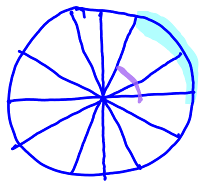
1. A central angle of $\pi/2$ intercepts an arc on the surface of the earth that runs from the equator to the North Pole. Using 3950 miles as the radius of the earth, find the length of the intercepted arc to the nearest mile.



$\cancel{2\pi}\ 3950\left(\frac{\frac{\pi}{2}}{\cancel{2\pi}}\right)$

6205 mi

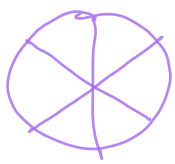
2. A wagon wheel has a diameter of 28 inches and an angle of 30° between the spokes. What is the length of the arc *s* (to the nearest hundredth of an inch) between two adjacent spokes?



$2\pi 14\left(\frac{60°}{360°}\right)$

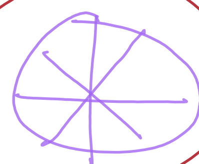
14.66in

3. Which is bigger: a slice of pizza from a 10″ diameter pizza cut into 6 slices, or a slice from a 12″ diameter pizza cut into 8 slices?



$\pi 5^2\left(\frac{1}{6}\right)$

13.1 in^2



$\pi 6^2\left(\frac{1}{8}\right)$

14.1 in^2

12" is bigger

4. A center-pivot irrigation system is used to water a circular field with radius 200 feet. In three hours the system waters a sector with a central angle of $\pi/8$. What area (in square feet) is watered in that time?

Velocity: The rate at which the location of an object is changing with respect to time.

Angular Velocity: The rate at which the angle is changing. If a point is in motion on a circle through an angle of α radians in time t, then its angular velocity ω is given by $\omega = \frac{\alpha}{t}$.

Angular velocity is usually expressed as radians per unit of time (radians/hr, radians/min, radians/sec, etc.)

Examples:

1. Convert 650 rpm (revolutions per minute) to radians per minute.
(Use the fact that 1 revolution = 2π radians).

$$\frac{650 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{4084.1 \text{ rad}}{\text{min}}$$

2. Convert the angular velocity of 1600 rad/hr to rad/sec.

$$\frac{1600 \text{rad}}{\text{hr}} \cdot \frac{\text{hr}}{3600 \text{ sec}} = \frac{.44 \text{ rad}}{\text{sec}}$$

3. A 24-inch lawnmower blade rotates at a rate of 2000 rpm. What is the angular velocity in radians per second of a point on the tip of the blade?

$$\frac{2000\text{ rev}}{\text{min}} \cdot \frac{\text{min}}{60\text{ sec}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} = \frac{209.4\text{ rad}}{\text{sec}}.$$

4. Find the angular velocity in radians per second for a particle that is moving in a circular path at 4 revolutions per second on a circle of radius 9 ft.

Linear Velocity: The rate at which the distance is changing. If a point is in motion on a circle of radius r through an angle of α radians in time t, then its linear velocity v is given by $v = \frac{s}{t}$, where s is the arc length determined by $s = \alpha r$.

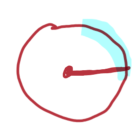
rev = circumference

Examples:

1. A propeller with a radius of 1.6 meters is rotating at 1500 revolutions per minute. What is the linear velocity in meters per minute for a point on the tip of the propeller?

$$\frac{1500\,\cancel{\text{rev}}}{\text{min}} \cdot \frac{2\pi\ 1.6\text{m}}{\cancel{1\text{ rev}}} = \frac{15079.6\text{ m}}{\text{min}}$$

2. Find the angular velocity in radians per second for a particle that is moving along a circle with diameter 15 meters at a linear velocity of 20 meters per second. radian = radius



$$\frac{20\text{m}}{\text{sec}} \cdot \frac{\text{rad}}{7.5\text{ m}} = \frac{2.67\text{ rad}}{\text{sec}}$$

3. Find the linear velocity in meters per second for a particle that is moving in a circular path at 7 revolutions per second on a circle of radius 15 meters.

4. What is the linear velocity in miles per hour of the tip of a 20-inch lawnmower blade that is rotating at 3000 rpm?

5. Find the linear velocity in miles per hour for a particle that is moving in a circular path at 1800 revolutions per minute on a circle with a diameter of 14 inches.

6. Find the angular velocity in radians per minute for a particle that is moving in a circular path at 95 mph on a circle with a radius of 8 inches.

Linear Velocity in Terms of Angular Velocity: If v is the linear velocity of a point on a circle of radius r, and ω is its angular velocity, then $v = r\omega$.

Example:
Any point on the surface of the earth (except at the poles) makes one revolution (2π radians) about the axis of the earth in 24 hours. So the angular velocity of a point on the earth is $2\pi/24$ or $\pi/12$ radians per hour. The linear velocity of a point on the surface of the earth depends on its distance from the axis of the earth. What is the linear velocity in miles per hour of a point on the equator? (Use 3950 miles as the radius of the earth).