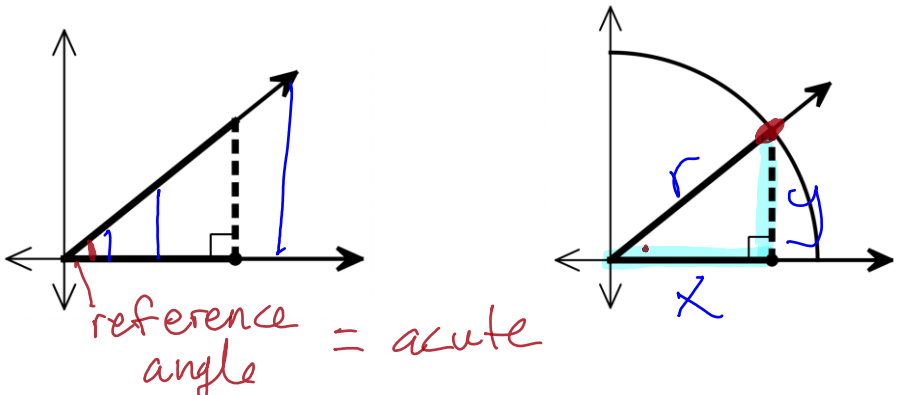
SM 3H

Date: 2/1/24 Section: 6.4

Objective: I can draw reference angles to find sides and angles.

Now that you know how to solve for a right triangle using right triangle trigonometry, let's put the right triangle in standard position and solve for all the parts of the triangle.

Reminder: Standard position: put triangle on x-y-plane



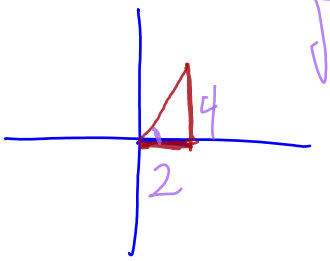
If (x, y) is any point other than the origin on the terminal side of an angle α in standard position and $r = \sqrt{x^2 + y^2}$.

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \qquad \cos\alpha = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \qquad \tan\alpha = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

$$\csc\alpha = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} \qquad \sec\alpha = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} \qquad \cot\alpha = \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$$

Examples:

Find the values of the six trigonometric functions of the angle α in standard position whose terminal side passes through $(2, 4)$.



$\sqrt{2^2+4^2} = 2\sqrt{5}$

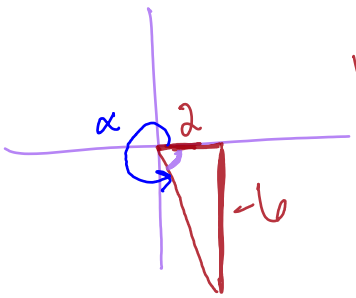
$\frac{4}{2\sqrt{5}}$

$\sin\alpha = \frac{2}{\sqrt{5}}$ $\csc\alpha = \frac{\sqrt{5}}{2}$

$\cos\alpha = \frac{1}{\sqrt{5}}$ $\sec\alpha = \sqrt{5}$

$\tan\alpha = 2$ $\cot\alpha = \frac{1}{2}$

Find the values of the six trigonometric functions of the angle α in standard position whose terminal side passes through $(2, -6)$.



$\sqrt{2^2+6^2} = 2\sqrt{10}$

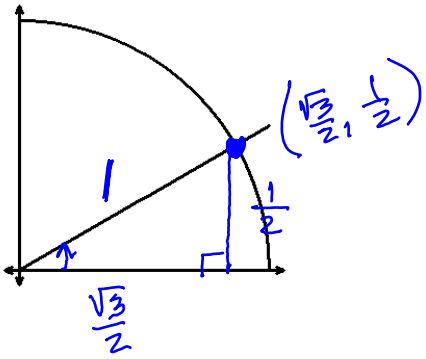
$\sin\alpha = \frac{-3}{\sqrt{10}}$ $\csc\alpha = \frac{\sqrt{10}}{-3}$

$\cos\alpha = \frac{1}{\sqrt{10}}$ $\sec\alpha = \sqrt{10}$

$\tan\alpha = -3$ $\cot\alpha = -\frac{1}{3}$

If the triangle is a special right triangle, then it is an important angle that we must know how to find and use when it is standard position.

Make a triangle with the picture to the right so the central angle is in standard position. If the radius is 1 and it is a special right triangle, find the missing sides.

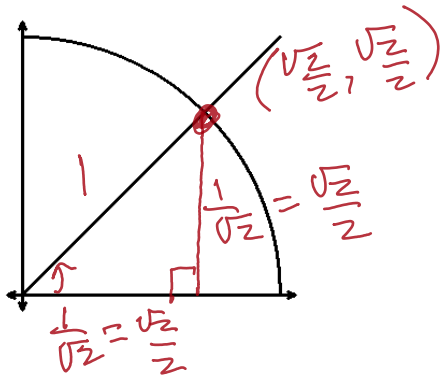


What is the standard position angle?

$\alpha = 30°$

$\text{ref}\angle = 30°$

Make a triangle with the picture to the right so the central angle is in standard position. If the radius is 1 and it is a special right triangle, find the missing sides.

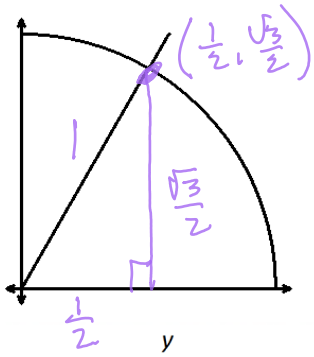


What is the standard position angle?

$\alpha = 45°$

$\text{ref}\angle = 45°$

Make a triangle with the picture to the right so the central angle is in standard position. If the radius is 1 and it is a special right triangle, find the missing sides.

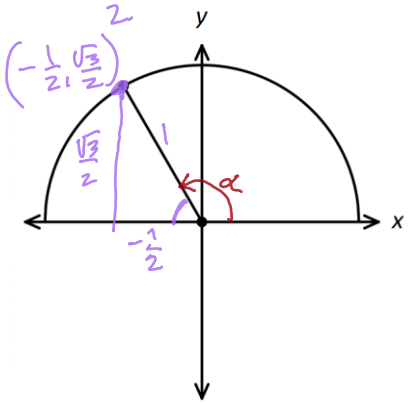


What is the standard position angle?

$\alpha = 60°$

$\text{ref}\angle = 60°$

Make a triangle with the picture to the right so the central angle is in standard position. If the radius is 1 and it is a special right triangle, find the missing sides.



What is the standard position angle?

$\alpha = 120°$

$\text{ref}\angle = 60°$

What is the angle of the triangle you are using in the triangle or the central angle?

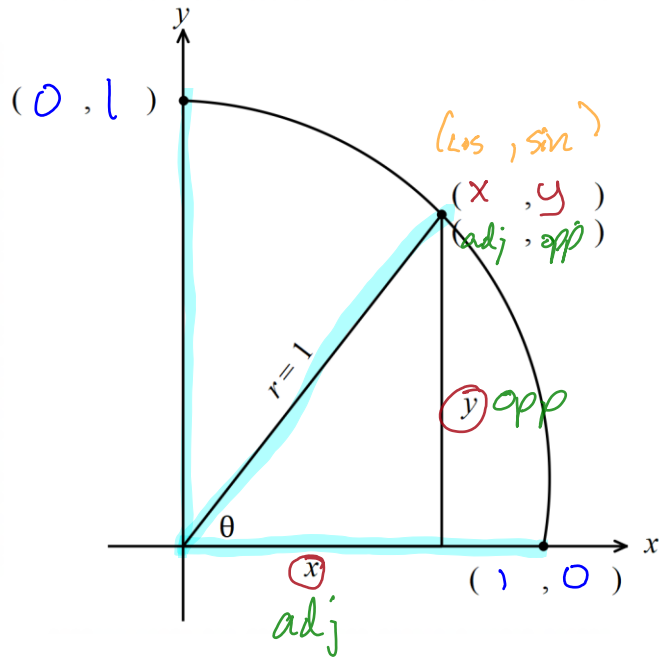
$60°$

This angle (the central angle of the triangle) is called the reference angle.

DO NOT MEMORIZE THE UNIT CIRCLE! DRAW THE REFERENCE ANGLE EVERY TIME!

Fill in the ordered pairs in the picture to the right.

The point where the triangle meets the circle has 2 ordered pairs. Why?



On the Unit Circle:

$\sin\alpha = y$ $\quad \cos\alpha = x$ $\quad \tan\alpha = \frac{y}{x} = \frac{\sin\alpha}{\cos\alpha}$

$\csc\alpha = \frac{1}{y}$ $\quad \sec\alpha = \frac{1}{x}$ $\quad \cot\alpha = \frac{x}{y} = \frac{\cos\alpha}{\sin\alpha}$

The *signs* of the trigonometric functions depend on the quadrant in which the angle lies and the corresponding signs of *x* and *y* (remember *r* is always positive).

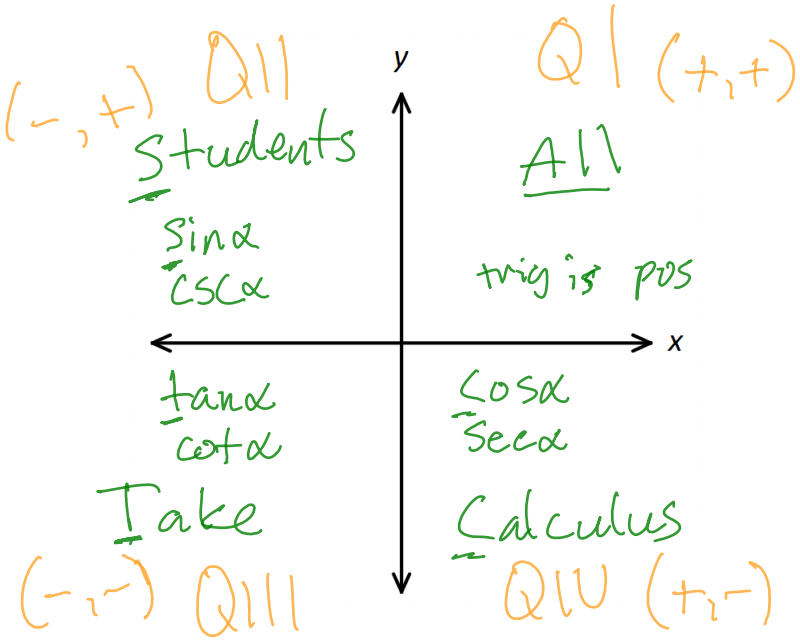
A good mnemonic to remember which functions are positive in each quadrant is "**All** **S**tudents **T**ake **C**alculus".

Quadrant I:

Quadrant II:

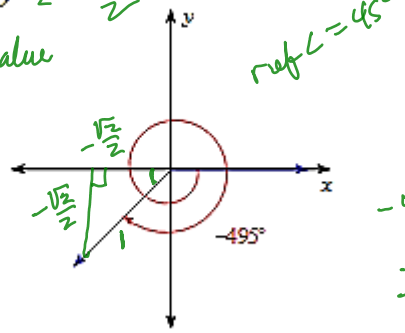
Quadrant III:

Quadrant IV:

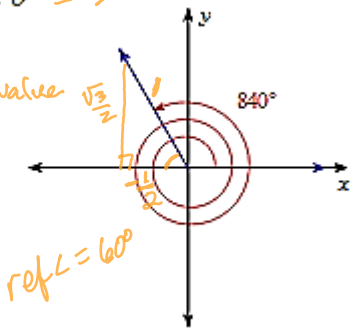


Examples: Find the exact value of each trig function by drawing a reference triangle.

1. $\sin\theta = -\frac{\sqrt{2}}{2}$

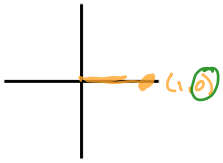


2. $\sec\theta = -2$

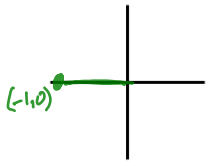


Examples: Find the exact values of the following. Draw a picture for your work. DO NOT use a unit circle.

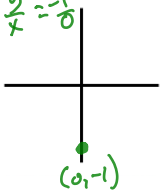
1. $\sin 0°$



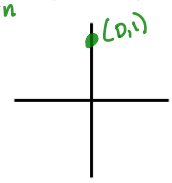
2. $\cos\pi$



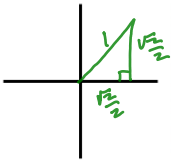
3. $\tan(-\pi/2)$ = undef



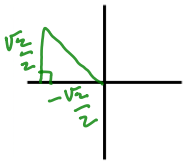
4. $\csc(-270°)$



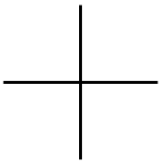
5. $\sin(\pi/4)$



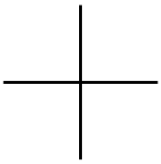
6. $\cos(-225°)$



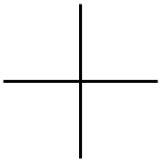
7. $\cot(5\pi/4)$



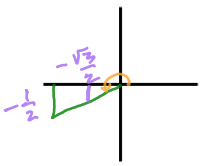
8. $\sec 315°$



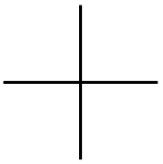
9. $\sin 30°$



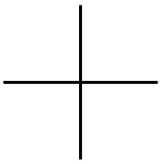
10. $\cos(7\pi/6)$



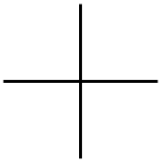
11. $\tan(-\pi/3)$



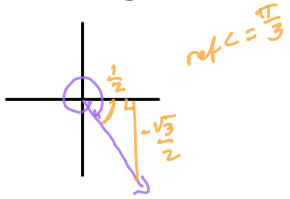
12. $\csc 150°$



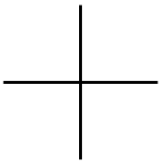
13. $\tan\frac{9\pi}{2}$



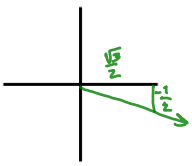
14. $\cos -\frac{7\pi}{3}$



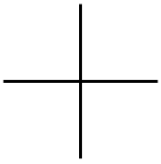
15. $\cot(-240°)$



16. $\sec(-\pi/6)$



17. $\cos(5\pi/3)$



17. $\tan(-150°)$