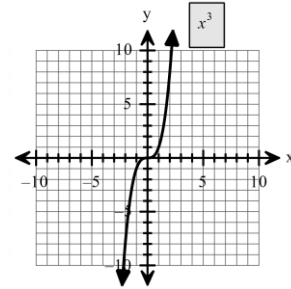


In order for a function to have an inverse, the function must be one-to-one. In other words, it must pass the _____ and _____ line test.

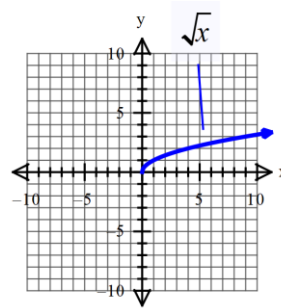
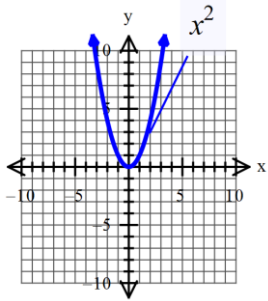
Is the function $f(x) = x^3$, one-to-one?

Does it have an inverse?



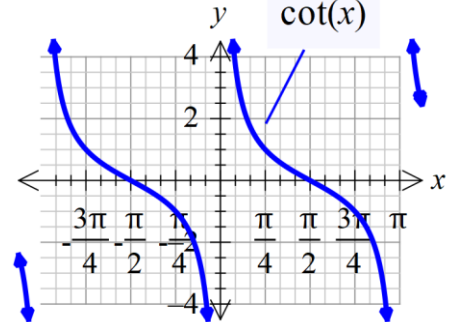
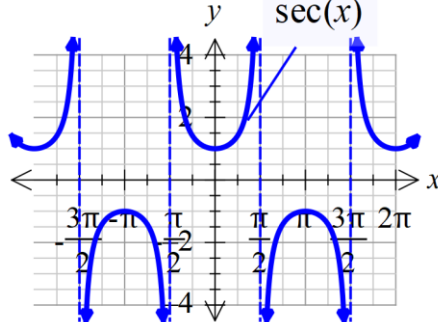
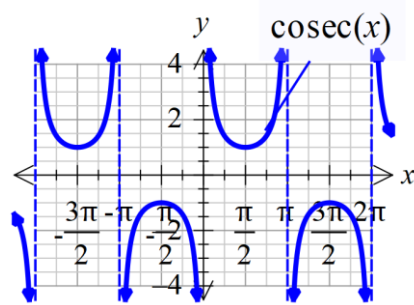
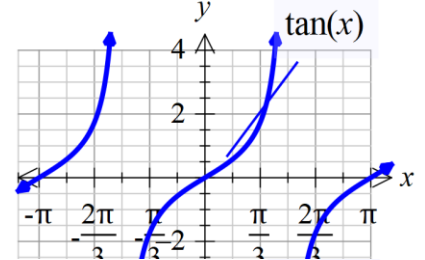
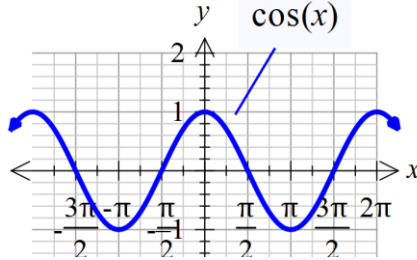
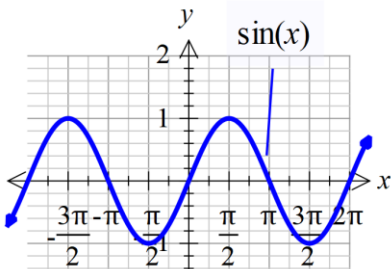
Is there a function that is not one-to-one? _____

What is the inverse of this function? _____



What do we need to do to make these one-to-one so they are true inverses? _____ the _____ of the quadratic function. So instead of _____, we restrict the domain of the square root function to _____. Therefore, the inverse function or square root function will have a restricted _____.

This is what the parent graphs of the trig functions look like:



Are the trig functions one-to-one?
How do we make them one-to-one?

$\sin^{-1}(x)$ is the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is x . Notice that this function on the calculator never goes above 90° and can also give you negative angles.

$\cos^{-1}(x)$ is the angle in $[0, \pi]$ whose cosine is x . Notice that this function on the calculator never goes above 180° and will never give you negative angles.

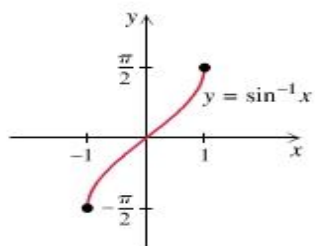
$\tan^{-1}(x)$ is the angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x . Notice that this function on the calculator never goes to 90° and can also give you negative angles.

Inverses of the Reciprocal Trigonometric Functions

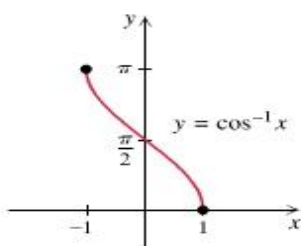
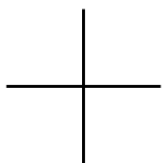
$\csc^{-1}(x)$ is the angle in $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ whose cosecant is x .

$\sec^{-1}(x)$ is the angle in $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ whose secant is x .

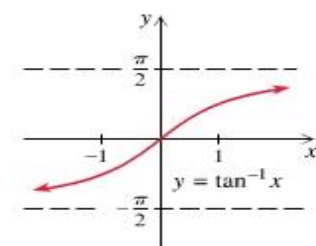
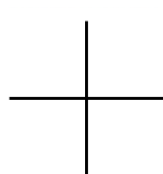
$\cot^{-1}(x)$ is the angle in $(0, \pi)$ whose cotangent is x .



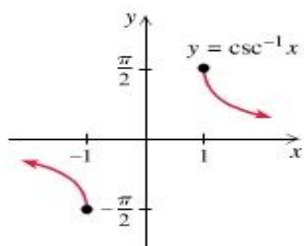
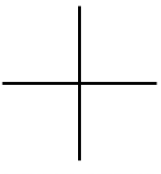
Domain $[-1, 1]$
Range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



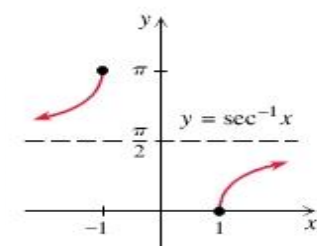
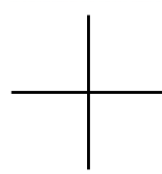
Domain $[-1, 1]$
Range $[0, \pi]$



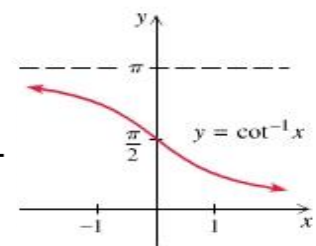
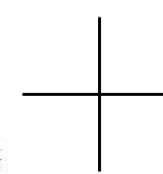
Domain $(-\infty, \infty)$
Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



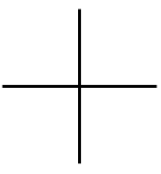
Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



Domain $(-\infty, \infty)$
Range $(0, \pi)$



Now are they one-to-one?

The **inverse sine function** is sometimes called the _____, and is abbreviated $\arcsin(x)$ or $\sin^{-1}(x)$.

Similarly, the other inverse functions are often called the _____ and _____, abbreviated $\arccos(x)$ or $\cos^{-1}(x)$ and $\arctan(x)$ or $\tan^{-1}x$.

Remember that the calculator does not do the reciprocal trig functions. To find $\csc(x)$, you have to type in $\frac{1}{\sin(x)}$. It is the same with the other two. This will give you the ratio of the sides. If you want to find the angle you need to use one of the identities below. These basically remind you to flip the fraction or side over (do opposite of SOH-CAH-TOA) when doing the inverse to find the angle.

$$\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$$

$$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$$

$$\cot^{-1}(x) = \begin{cases} \tan^{-1}(1/x) & \text{for } x > 0 \\ \tan^{-1}(1/x) + \pi & \text{for } x < 0 \\ \pi/2 & \text{for } x = 0 \end{cases}$$

$$\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$$

Example: Find the exact value of each expression without using a table or calculator. Work is to draw a triangle.

a) $\sin^{-1}\left(\frac{1}{2}\right)$

b) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

c) $\tan^{-1}(1)$

d) $\arccos\left(-\frac{1}{2}\right)$

Example: Find the angle α .

a) $\sin \alpha = 0.56, -90^\circ \leq \alpha \leq 90^\circ$

b) $\tan \alpha = -3, -\pi/2 < \alpha < \pi/2$

c) $\cos \alpha = 0.23, 0^\circ \leq \alpha \leq 180^\circ$

d) $\cos \alpha = -0.82, 0 \leq \alpha \leq \pi$

Example: Find the exact value of each expression without using a table or calculator. Draw a triangle.

a) $\operatorname{arcsec}(-2)$ b) $\operatorname{csc}^{-1}(2)$ c) $\operatorname{arccot}\left(\frac{1}{\sqrt{3}}\right)$

Example: Find the approximate value of each expression rounded to 4 decimal places.

a) $\operatorname{arccsc}(-1.4713)$ b) $\cot^{-1}(-2.5)$ c) $\sec^{-1}(4.328)$

Example: Find the exact value of each composition. Draw triangles.

a) $\sin\left(\cot^{-1}(-1)\right)$ b) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$ c) $\arcsin\left(\cos\left(\frac{\pi}{6}\right)\right)$

Examples: Find the exact value of each composition. Draw triangles.

a) $\sin\left(\cos^{-1}\left(\frac{8}{17}\right)\right)$ b) $\cot\left(\arccos\left(\frac{5}{13}\right)\right)$

Example: Find an equivalent algebraic expression for $\sin(\arctan(x))$. Draw triangles.

Example: Find an equivalent algebraic expression for $\sin(\operatorname{arccot}(x))$. Draw triangles.