

Date: 2/7/24

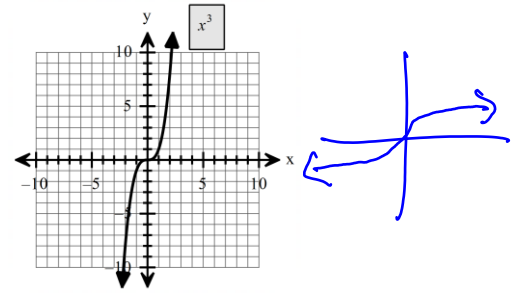
Section: 6.5

Objective: I can evaluate inverse trig and composition of trig functions.

In order for a function to have an inverse, the function must be one-to-one. In other words, it must pass the vertical and horizontal line test.

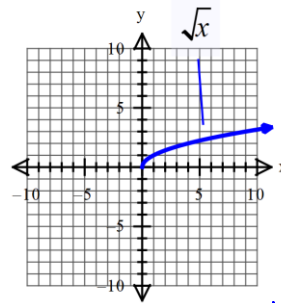
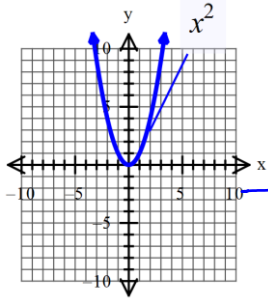
Is the function $f(x) = x^3$, one-to-one? yes

Does it have an inverse? $\sqrt[3]{x}$



Is there a function that is not one-to-one? quadratic

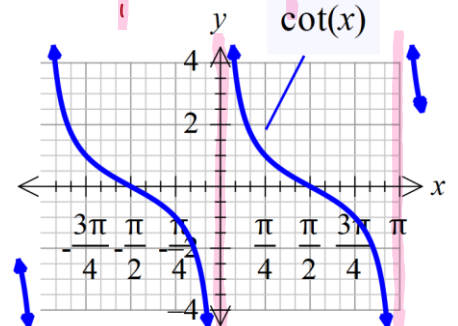
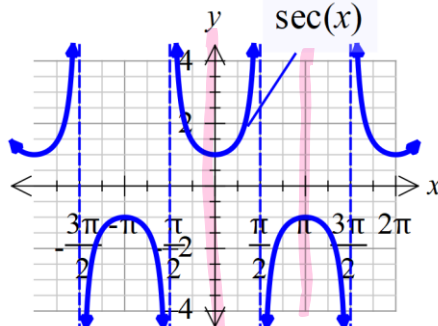
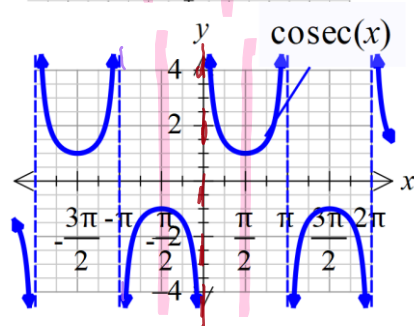
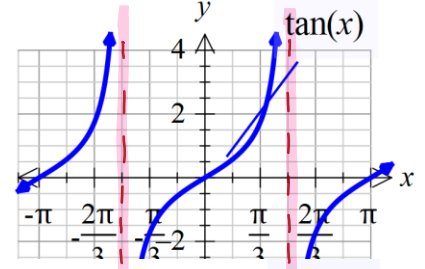
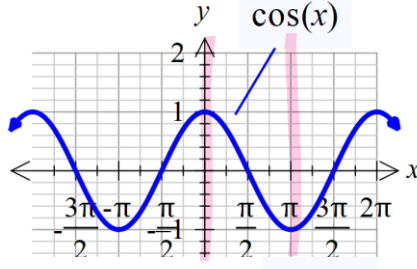
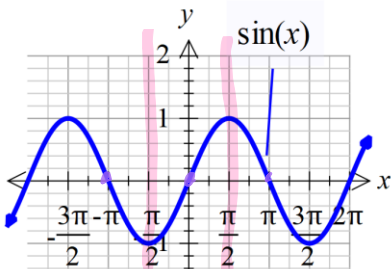
What is the inverse of this function? square root



cut off half
or restriction

What do we need to do to make these one-to-one so they are true inverses? Restrict the range of the quadratic function. So instead of $(-\infty, \infty)$, we restrict the domain of the square root function to $[0, \infty)$. Therefore, the inverse function or square root function will have a restricted domain.

This is what the parent graphs of the trig functions look like:



Are the trig functions one-to-one? no

How do we make them one-to-one? restricted the domain on inverse

$\sin^{-1}(x)$ is the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is x . Notice that this function on the calculator never goes above 90° and can also give you negative angles.

$\cos^{-1}(x)$ is the angle in $[0, \pi]$ whose cosine is x . Notice that this function on the calculator never goes above 180° and will never give you negative angles.

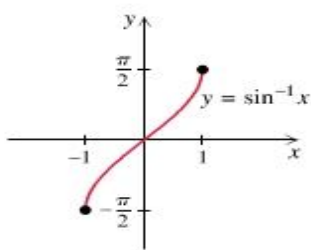
$\tan^{-1}(x)$ is the angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x . Notice that this function on the calculator never goes to 90° and can also give you negative angles.

Inverses of the Reciprocal Trigonometric Functions

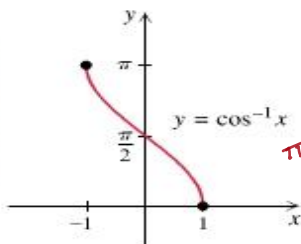
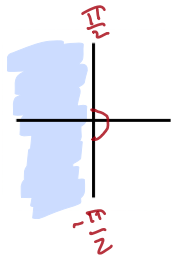
$\csc^{-1}(x)$ is the angle in $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ whose cosecant is x .

$\sec^{-1}(x)$ is the angle in $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ whose secant is x .

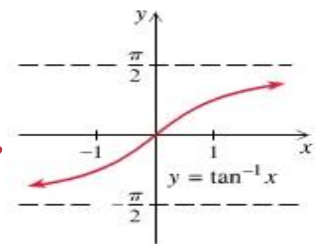
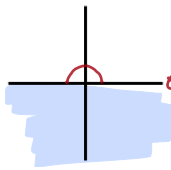
$\cot^{-1}(x)$ is the angle in $(0, \pi)$ whose cotangent is x .



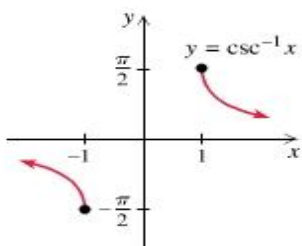
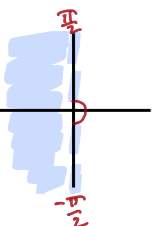
Domain $[-1, 1]$
Range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



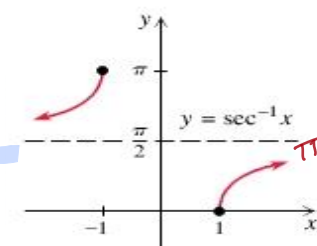
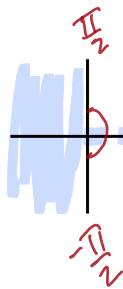
Domain $[-1, 1]$
Range $[0, \pi]$



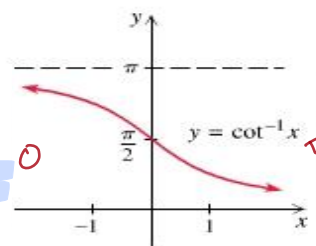
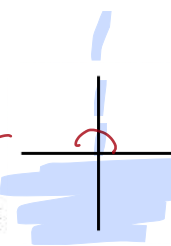
Domain $(-\infty, \infty)$
Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



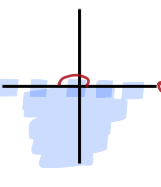
Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



Domain $(-\infty, \infty)$
Range $(0, \pi)$



Now are they one-to-one?

The **inverse sine function** is sometimes called the arc sine, and is abbreviated $\arcsin(x)$ or $\sin^{-1}(x)$.

Similarly, the other inverse functions are often called the arc cosine and arc tangent, abbreviated $\arccos(x)$ or $\cos^{-1}(x)$ and $\arctan(x)$ or $\tan^{-1}x$.

Remember that the calculator does not do the reciprocal trig functions. To find $\csc(x)$, you have to type in $\frac{1}{\sin(x)}$. It is the same with the other two. This will give you the ratio of the sides. If you want to find the angle you need to use one of the identities below. These basically remind you to flip the fraction or side over (do opposite of SOH-CAH-TOA) when doing the inverse to find the angle.

$$\csc^{-1}\left(\frac{x}{1}\right) = \sin^{-1}\left(\frac{1}{x}\right) = \alpha$$

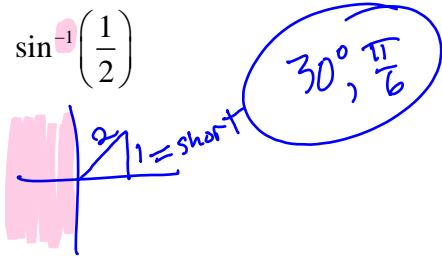
$$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$$

$$\cot^{-1}(x) = \begin{cases} \tan^{-1}(1/x) & \text{for } x > 0 \\ \tan^{-1}(1/x) + \pi & \text{for } x < 0 \\ \pi/2 & \text{for } x = 0 \end{cases}$$

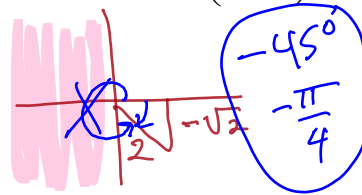
$$\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$$

Example: Find the exact value of each expression without using a table or calculator. Work is to draw a triangle.

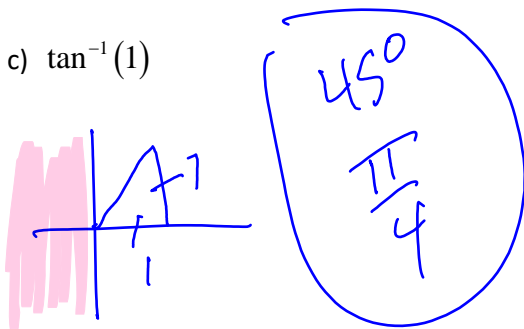
a) $\sin^{-1}\left(\frac{1}{2}\right)$



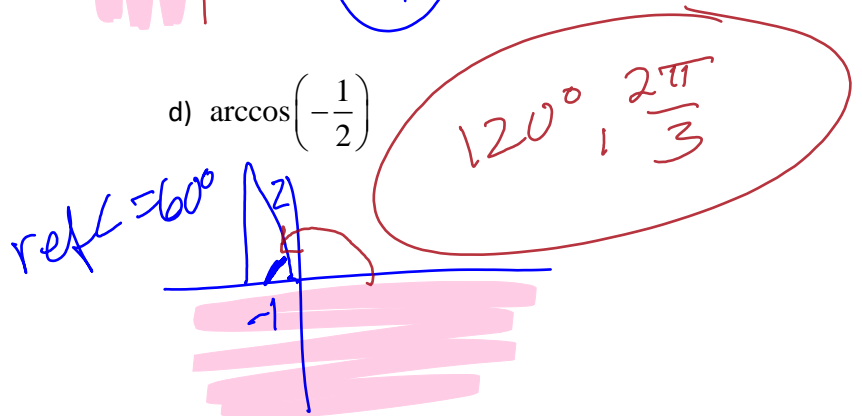
b) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$



c) $\tan^{-1}(1)$

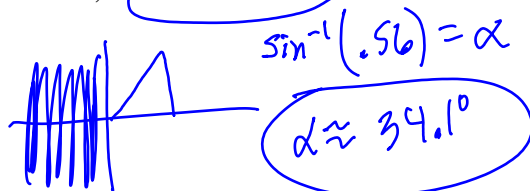


d) $\arccos\left(-\frac{1}{2}\right)$

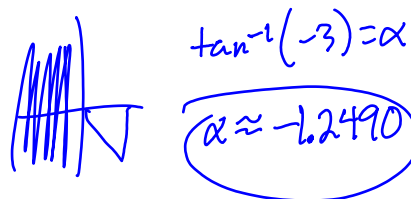


Example: Find the angle α

a) $\sin \alpha = 0.56, -90^\circ \leq \alpha \leq 90^\circ$



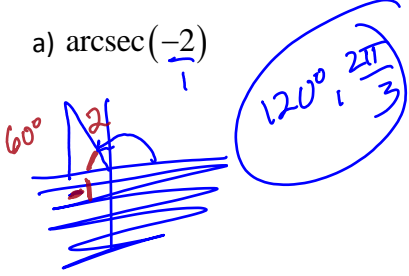
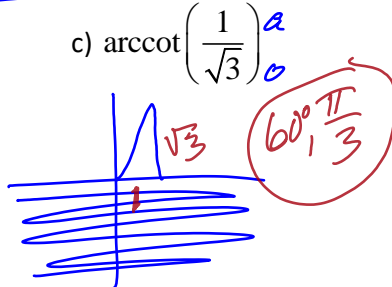
b) $\tan \alpha = -3, -\pi/2 < \alpha < \pi/2$



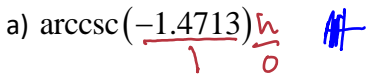
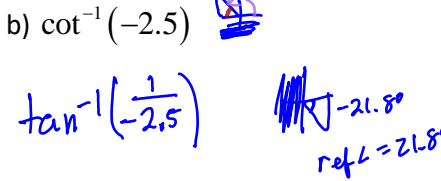
c) $\cos \alpha = 0.23, 0^\circ \leq \alpha \leq 180^\circ$

d) $\cos \alpha = -0.82, 0 \leq \alpha \leq \pi$

Example: Find the exact value of each expression without using a table or calculator. Draw a triangle.

a) $\operatorname{arcsec}(-2)$  b) $\csc^{-1}(2)$ c) $\operatorname{arccot}\left(\frac{1}{\sqrt{3}}\right)$ 

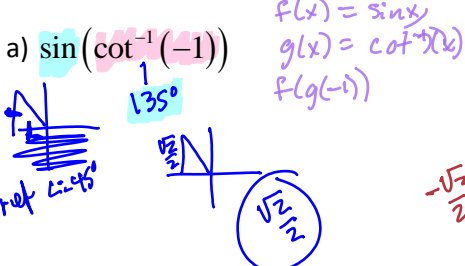
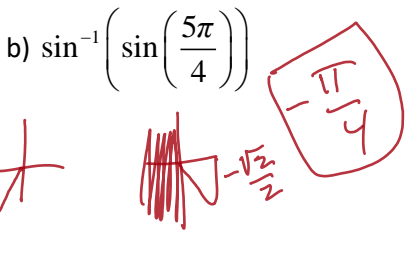
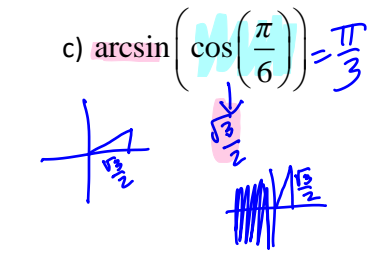
Example: Find the approximate value of each expression rounded to 4 decimal places.

a) $\operatorname{arccsc}\left(\frac{-1.4713}{1}\right)$  b) $\cot^{-1}(-2.5)$  c) $\sec^{-1}(4.328)$

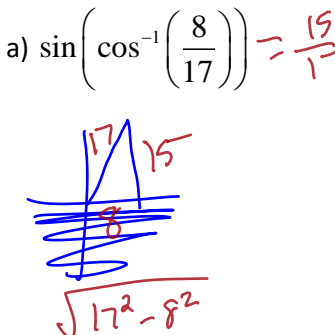
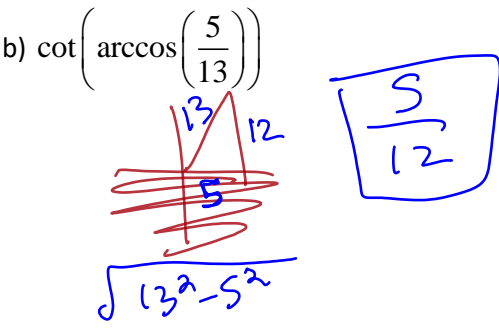
$\sin^{-1}\left(\frac{1}{-1.4713}\right)$ $\alpha \approx -.7473$

$\tan^{-1}\left(-\frac{1}{2.5}\right)$ $180^\circ - 21.8^\circ = 158.2^\circ$

Example: Find the exact value of each composition. Draw triangles.

a) $\sin(\cot^{-1}(-1))$  b) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$  c) $\arcsin\left(\cos\left(\frac{\pi}{6}\right)\right)$ 

Examples: Find the exact value of each composition. Draw triangles.


a) $\sin\left(\cos^{-1}\left(\frac{8}{17}\right)\right) = \frac{15}{17}$  b) $\cot\left(\arccos\left(\frac{5}{13}\right)\right)$ 

Example: Find an equivalent algebraic expression for $\sin(\arctan(x))$. Draw triangles.



$\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}$

Example: Find an equivalent algebraic expression for $\sin(\operatorname{arccot}(x))$. Draw triangles.



$\frac{1}{\sqrt{x^2+1}}$