SM 3H

Date: 2/26/24 **Section:** 7.2

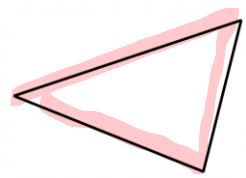
Objective: I can find sides and angles of a triangle using law of cosines. I can find area of a triangle.

What if an oblique triangle is not a ASA, AAS, or SSA triangle? What do we use instead?

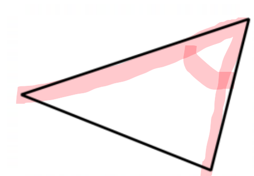
Law of Cosines

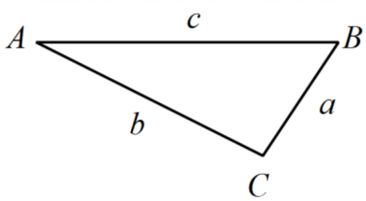
★ When do you use Law of Cosines?

SSS triangle



SAS triangle





Law of Cosines:

Solve for the ***largest*** side or angle first!!!!!!!

$a^2 = b^2 + c^2 - 2bc\cos A$

OR

$b^2 = a^2 + c^2 - 2ac\cos B$

OR

$c^2 = a^2 + b^2 - 2ab\cos C$

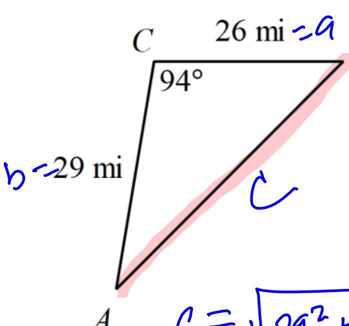
$\sin^{-1}$

$\cos^{-1}$

SSS: Use the fact that the largest angle is across from the longest side of the triangle to solve for the largest angle using the law of cosines. (For example, if *c* is the longest side, use the equation $c^2 = a^2 + b^2 - 2ab\cos\gamma$ to solve for γ.) Then use the law of sines to find the remaining angles, which will both be acute. **Don't use the law of sines to solve for any angle that might be obtuse! The law of sines will always give you acute angle measures!**

Example:

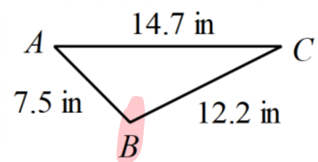
a. Find AB SAS



$c = \sqrt{29^2 + 26^2 - 2(29)(26)\cos 94°}$

$c \approx 40.3$ mi

b. Find $m\angle B$ SSS



$14.7^2 = \cancel{7.5^2} + \cancel{12.2^2} - 2(7.5)(12.2)\cos B$

$-7.5^2 \quad -7.5^2 \quad -12.2^2$

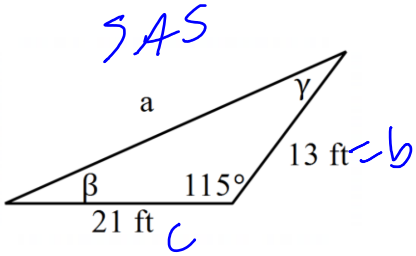
-12.2^2

$$\frac{14.7^2 - 7.5^2 - 12.2^2 = -2(7.5)(12.2)\cos B}{-2(7.5)(12.2)}$$

$$\cos^{-1}\left(\frac{14.7^2 - 7.5^2 - 12.2^2}{-2(7.5)(12.2)}\right) = B$$

$B \approx 93.4°$

Example: Solve each triangle. Round your answers to the nearest tenth.



$m\angle\alpha =$ 115° a = 29.0 ft

$m\angle\beta =$ 24.0° b = 13 ft

$m\angle\gamma =$ 41.0° c = 21 ft

$a = \sqrt{21^2 + 13^2 - 2(21)(13)\cos 115°}$

$13^2 = 29^2 + 21^2 - 2(29)(21)\cos\beta$

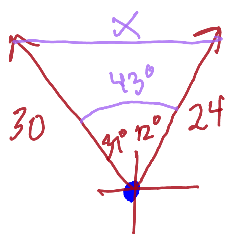
$\cos^{-1}\left(\frac{13^2 - 29^2 - 21^2}{-2(29)(21)}\right) = B$

OR $\left(\frac{\sin\beta}{13} = \frac{\sin 115°}{29.0}\right)13$

$\sin^{-1}\left(\frac{13\sin 115°}{29.0}\right) = B$

Example: Jan and Dean started hiking from the same location at the same time. Jan hiked at 4 mph with bearing N12°E, and Dean hiked at 5 mph with bearing N31°W. How far apart were they after 6 hours? Round to the nearest tenth of a mile.

x = distance between friends

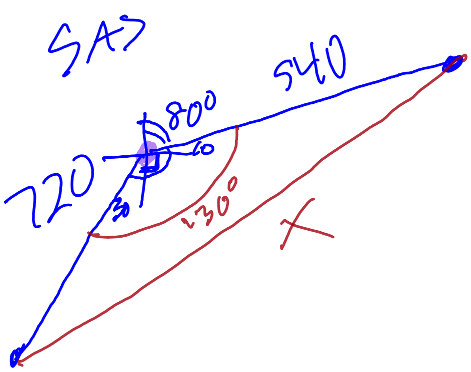


$x = \sqrt{24^2 + 30^2 - 2(24)(30)\cos 43°}$

$x \approx 20.6$ mi

Example: Ms. Peterson and Ms. Gordon left the airport at the same time. Ms. Peterson flew at 180 mph on a course with bearing 80 degrees, and Ms. Gordon flew at 240 mph on a course with bearing 210 degrees. How far apart were they after 3 hours? Round to the nearest tenth of a mile.

= x



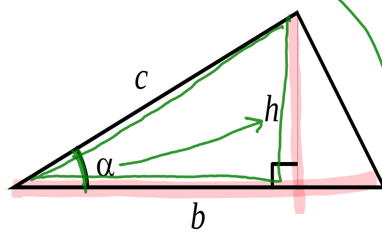
$x = \sqrt{720^2 + 540^2 - 2(720)(540)\cos 130°}$

$x \approx 1144.5$ mi

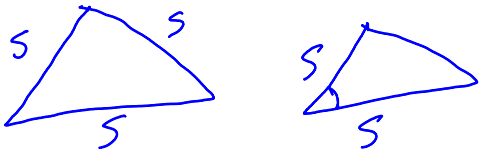
Area of an Oblique Triangle

The formula $\frac{1}{2}bh$ gives the area of a triangle

if you are given the base and the height.



What is another way we can write the height using other sides and angles?



$c\left(\sin\alpha = \frac{h}{c}\right)$

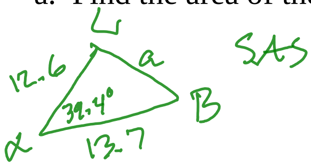
$h = c\sin\alpha$

Using substitution, we derive the formula $\frac{1}{2}bc\sin\alpha$, which you use with a SAS triangle.

Depending on which angles and sides are known, the formulas $\frac{1}{2}ab\sin C$ and $\frac{1}{2}ac\sin B$ can also be used.

Examples:

a. Find the area of the triangle with $\alpha = 39.4^\circ$, $b = 12.6$, and $c = 13.7$

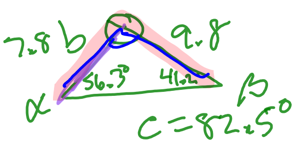


SAS

$\frac{1}{2}(12.6)(13.7)\sin 39.4^\circ$

$A \approx 54.8$ sq units

b. Find the area of a triangle with $\alpha = 56.3^\circ$, $\beta = 41.2^\circ$, and $a = 9.8$



$C = 82.5^\circ$

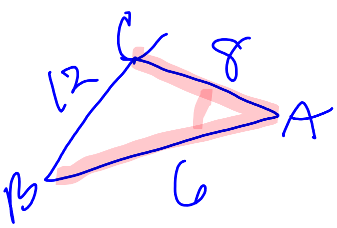
$\left(\frac{b}{\sin 41.2^\circ} = \frac{9.8}{\sin 56.3^\circ}\right)\sin 41.2^\circ$

$b \approx 7.8$

$\frac{1}{2}(7.8)(9.8)\sin 82.5^\circ$

$A \approx 37.9$ sq units

c. Find the area of the triangle with $a = 12$, $b = 8$, and $c = 6$



$12^2 = 8^2 + 6^2 - 2(6)(8)\cos A$

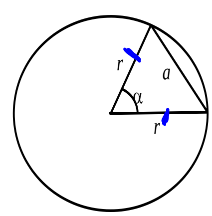
$\cos^{-1}\left(\frac{144-64-36}{-96}\right) = A$

$A \approx 117.3^\circ$

$\frac{1}{2}(8)(6)(\sin 117.3^\circ)$

$A = 21.3$ sq units

Length of a Chord: If a chord of length a is intercepted by a central angle α in a circle of radius r, then $a = r\sqrt{2-2\cos\alpha}$. (This formula is derived from the law of cosines.)



Example: Find the length of the chord intercepted by a central angle of 33.8° in a circle of radius 22.4 ft.

$a = \sqrt{r^2 + r^2 - 2 \cdot r \cdot r\cos\alpha}$

$a = \sqrt{2r^2 - 2r^2\cos\alpha}$

$a = \sqrt{r^2(2-2\cos\alpha)}$

$a = r\sqrt{2-2\cos\alpha}$