SM 3H

Date: 4/2/24 **Section:** 8.1

Objective: I can use basic trig identities to simplify expressions.

Identities you know (see notecards for details):

1. Right triangle trig (SOH-CAH-TOA) Use the letters a, o, h.

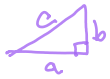
2. Right triangle trig in standard position. Use the letters x, y, r.

3. Reciprocal functions

4. $\tan\theta$ and $\cot\theta$ written in terms of sine and cosine.

New identities (can see notecards):

1. Pythagorean Identities Pythagorean Theorem $a^2 + b^2 = c^2$



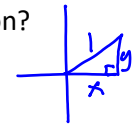
What letters can we replace a, b, and c with if the triangle is in standard position?

What trig function represents x? $\cos\theta$

What trig function represents y? $\sin\theta$

What is the hypotenuse on a unit circle? 1

$x^2 + y^2 = r^2$



Now rewrite the Pythagorean theorem using the information above.

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$ memorize

We can also rewrite this using the reciprocal functions.

Using cosecant: $\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\cot^2\theta + 1 = \csc^2\theta$

Mnemonic device: 1 cott age cheese

Using secant: $\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1 + \tan^2\theta = \sec^2\theta$

Mnemonic device: 1 ting second

2. Even/odd functions 

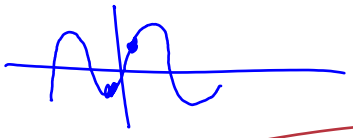
What are the even trig functions? $\cos\theta$, $\sec\theta$

What is the output if we use the positive and negative input when it is even?

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ 

What are the odd trig functions? $\sin\theta$, $\tan\theta$, $\cot\theta$, $\csc\theta$

What is the output if we use the positive and negative input when it is odd?



$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ 

Even identities:

$\cos\theta = \cos(-\theta)$

$\sec\theta = \sec(-\theta)$

Odd identities:

$-\sin\theta = \sin(-\theta)$

$-\tan\theta = \tan(-\theta)$

$-\cot\theta = \cot(-\theta)$

$-\csc\theta = \csc(-\theta)$

memorize

When simplifying trig identities there is no specific pattern to follow. **AND** there could be multiple ways to do it.

But here are some hints to use:

1. change everything to sine and cosine
2. rewrite using a Pythagorean identity $\sin^2\theta + \cos^2\theta$ 1
3. Find a common denominator
4. rewrite an identity in terms of another by manipulating the identity

Using the Pythagorean Identity, write it as $\cos^2 x$.

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

Rewrite the Pythagorean Identity as $\tan^2 x$. $\tan^2 x = \sec^2 x - 1$

$1 + \tan^2 x = \sec^2 x$

Now use that version of the identity to rewrite $\cot^2 x$ in terms of cosecant only.

$1 + \cot^2 x = \csc^2 x$ $\cot^2 x = \csc^2 x - 1$

Examples: Simplify to one trig function or number.

1. $\dfrac{\tan x}{\sec x}$

$\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}}$

$\sin x$

2. $\dfrac{\tan x \csc x}{\sec x}$

(1) $\dfrac{\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}}{\frac{1}{\cos x}}$

(2) $\dfrac{\frac{1}{\cos x}}{\frac{1}{\cos x}}$

(3) 1

OR

(1) $\dfrac{\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}}{\sec x}$

(2) $\dfrac{\frac{1}{\cos x}}{\sec x}$

(3) $\dfrac{\sec x}{\sec x}$

(4) 1

3. $\sin\alpha + \cot\alpha \cos\alpha$

(1) $\dfrac{\sin\alpha}{1} + \dfrac{\cos\alpha}{\sin\alpha} \cdot \dfrac{\cos\alpha}{1}$

(2) $\left(\dfrac{\sin\alpha}{\sin\alpha}\right)\dfrac{\sin\alpha}{1} + \dfrac{\cos^2\alpha}{\sin\alpha}$

(3) $\dfrac{\sin^2\alpha}{\sin\alpha} + \dfrac{\cos^2\alpha}{\sin\alpha}$

(4) $\dfrac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha}$

(5) $\dfrac{1}{\sin\alpha}$

(6) $\csc\alpha$

4. $\csc(-x)\tan(-x)$

(1) $-\csc x \cdot -\tan x$

(2) $\csc x \tan x$

(3) $\dfrac{1}{\sin x} \cdot \dfrac{\sin x}{\cos x}$

(4) $\dfrac{1}{\cos x}$

(5) $\sec x$

5. $\dfrac{1}{1+\cos(-\alpha)} + \dfrac{1}{1-\cos\alpha}$

(1) $\left(\dfrac{1-\cos\alpha}{1-\cos\alpha}\right)\dfrac{1}{1+\cos\alpha} + \dfrac{1}{1-\cos\alpha}\left(\dfrac{1+\cos\alpha}{1+\cos\alpha}\right)$

(2) $\dfrac{1-\cos\alpha}{1-\cos^2\alpha} + \dfrac{1+\cos\alpha}{1-\cos^2\alpha}$

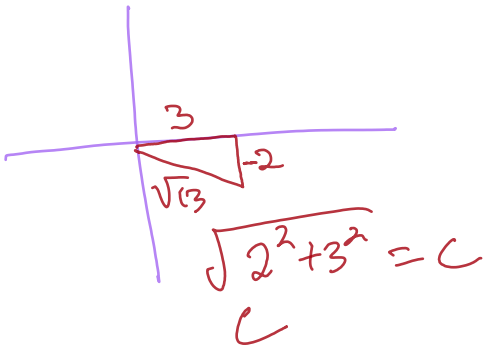
(3) $\dfrac{2}{1-\cos^2\alpha}$ $\quad \cos^2 x + \sin^2 x = 1$

(4) $\dfrac{2}{\sin^2\alpha}$

(5) $2\csc^2\alpha$

Example: Use the identities to find the function values. Draw a picture as part of your work. (Use Pythagorean triples, when possible.) 3 4 5 5 12 13

1. If $\tan\alpha = -\frac{2}{3}$ and α is in quadrant IV, find the values of the remaining five trigonometric functions.



$\sqrt{2^2+3^2} = c$

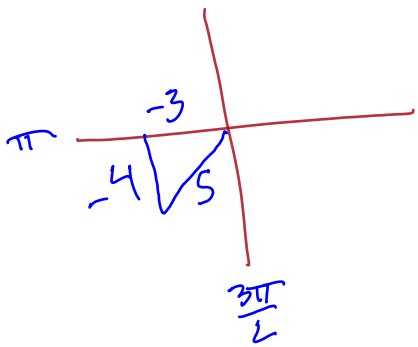
c

$\sin\alpha = \frac{-2}{\sqrt{13}}$ $\quad \csc\alpha = -\frac{\sqrt{13}}{2}$

$\cos\alpha = \frac{3}{\sqrt{13}}$ $\quad \sec\alpha = \frac{\sqrt{13}}{3}$

$\tan\alpha = -\frac{2}{3}$ $\quad \cot\alpha = -\frac{3}{2}$

2. If $\tan\theta = \frac{4}{3}$ and $\pi < \theta < \frac{3\pi}{2}$, find the values of the remaining five trigonometric functions.



$\sin\theta = -\frac{4}{5}$ $\quad \csc\theta = -\frac{5}{4}$

$\cos\theta = -\frac{3}{5}$ $\quad \sec\theta = -\frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\quad \cot\theta = \frac{3}{4}$