SM 3H

Date: 4/4/24 **Section:** 8.2

Objective: I can prove trig identities.

This is the hardest section of the entire year. You need to learn to think through a problem. There are multiple ways to get to the end of a proof. DO NOT just look at the answers, you will not learn how to think through the problem. It is also hard because of the algebra involved. Don't forget in a proof you MUST show every single step!

Hints:

1. DO NOT SKIP STEPS!
2. Change only one side at a time. When you get to the end of one side, then you can change the other side.
3. You cannot cross the equal sign! All of your work must be done on one side only.
4. Always write the equal sign in each step of your proof.
5. Change everything to sine and cosine.
6. Multiply expression together or factor the expression. Remember you can only do one side at a time.
7. Rewrite using a Pythagorean Identity.
8. Find common denominator so you can add or subtract. Or separate a fraction into 2 fractions with the same denominator.
9. Most proofs can be done in less than 10 steps. If it takes you more than 10 steps to prove, go back and start over because there is a faster way.

Review examples:

1. Factor.

$\sin^2 x + 3\sin x + 2$

$(\sin x + 2)(\sin x + 1)$

2. Multiply.

$(2\cos x - 3)(\cos x + 1)$

$2\cos^2 x - \cos x - 3$

3. Factor.

$\sec^2 x - \tan^2 x$

$(\sec x + \tan x)(\sec x - \tan x)$

4. Multiply.

$(2\sin x + 1)^2$

$4\sin^2 x + 4\sin x + 1$

Examples: Verify each identity. (This means prove so show all of your work no matter how small.)

1. $1 + \sec x \sin x \tan x = \sec^2 x$

(1) $1 + \frac{1}{\cos x} \cdot \frac{\sin x}{1} \cdot \frac{\sin x}{\cos x} =$

(2) $1 + \frac{\sin^2 x}{\cos^2 x} =$

(3) $1 + \tan^2 x =$

(4) $\sec^2 x = \sec^2 x$

2. $\frac{\cos \alpha}{1-\sin \alpha} = \frac{1+\sin \alpha}{\cos \alpha}$

(1) $\frac{\cos^2 \alpha}{(1-\sin\alpha)(\cos\alpha)} =$

(2) $\frac{1-\sin^2 \alpha}{(1-\sin\alpha)\cos\alpha} =$

(3) $\frac{(1+\sin\alpha)(1-\sin\alpha)}{(1-\sin\alpha)\cos\alpha} =$

(4) $\frac{1+\sin\alpha}{\cos\alpha} = \frac{1+\sin\alpha}{\cos\alpha}$

3. $\frac{\csc x - \sin x}{\sin x} = \cot^2 x$

(1) $\frac{\csc x}{\sin x} - \frac{\sin x}{\sin x} =$

(2) $\frac{\frac{1}{\sin x}}{\frac{\sin x}{1}} - 1 =$

(3) $\frac{1}{\sin x} \cdot \frac{1}{\sin x} - 1 =$

(4) $\frac{1}{\sin^2 x} - 1 =$

(5) $\csc^2 x - 1 =$

(6) $\cot^2 x = \cot^2 x$

OR

(1) $\frac{\frac{1}{\sin x} - \sin x \left(\frac{\sin x}{\sin x}\right)}{\sin x}$

(2) $\frac{\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}}{\sin x} =$

(3) $\frac{\frac{1-\sin^2 x}{\sin x}}{\sin x} =$

(4) $\frac{1-\sin^2 x}{\sin^2 x} =$

(5) $\frac{\cos^2 x}{\sin^2 x} =$

(6) $\cot^2 x = \cot^2 x$

4. $\frac{1-\cos^2(-t)}{\sin(-t)} = \tan(-t)\cos(-t)$

$1+\sin^2(-t) \quad (-\sin t)^2$

$1-\sin^2 t \quad (\sin(-t))^2$

$\frac{1-\cos^2 t}{-\sin t} =$

$\frac{\sin^2 t}{-\sin t} =$

$-\sin t = -\tan t \cos t$

$= -\frac{\sin t}{\cos t} \cdot \cos t$

$-\sin t = -\sin t$

5. $-2\cot^2 x = \frac{1}{1-\sec x} + \frac{1}{1+\sec x}$ $\left(\frac{1+\sec x}{1+\sec x}\right)\left(\frac{1-\sec x}{1-\sec x}\right)$

(1) $= \frac{1+\sec x}{(1+\sec x)(1-\sec x)} + \frac{1-\sec x}{(1+\sec x)(1-\sec x)}$

(2) $= \frac{2}{1-\sec^2 x}$

(3) $= \frac{2}{-(\sec^2 x - 1)}$

(4) $= \frac{-2}{\tan^2 x}$

(5) $-2\cot^2 x = -2\cot^2 x$

6. $\frac{1-\sin^2 t}{1-\csc(-t)} = \frac{1+\sin(-t)}{\csc t}$

(1) $\frac{(1-\sin t)(1+\sin t)}{1+\csc t} =$

(2) $\frac{(1-\sin t)(1+\sin t)}{1+\frac{1}{\sin t}} =$ $\left(\frac{\sin t}{\sin t}\right)$

(3) $\frac{(1-\sin t)(1+\sin t)}{\frac{\sin t}{\sin t} + \frac{1}{\sin t}} =$

(4) $\frac{(1-\sin t)(1+\sin t)}{\frac{\sin t+1}{\sin t}} =$

(5) $(1-\sin t)(1+\sin t)\left(\frac{\sin t}{\sin t+1}\right) =$

(6) $(1-\sin t)(\sin t) =$

(7) $\frac{1-\sin t}{\csc t} =$

(8) $\frac{1+\sin(-t)}{\csc t} = \frac{1+\sin(-t)}{\csc t}$