SM 3H

Date: 4/8/24 **Section:** 8.3

Objective: I can use sum and difference of cosine identity and cofunction identities.

Sometimes the angle you are trying to find the cosine of is not on the unit circle. When this happens, find two angles that are on the unit circle that either add or subtract to the angle you are trying to find. Then you can use the following identity.

Sum or Difference of Cosines Identity:.

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

Mnemonic device: Cosine change silly sign

First practice rewriting angles as a sum or difference of angles on the unit circle.

Examples: Write the following angles as sums or differences of two other angles whose trigonometric functions can be calculated exactly. (There can be more than one answer.)

1. $105°$
   $60° + 45°$
   $135° - 30°$
2. $15°$
   $45° - 30°$
   $60° - 45°$
3. $\frac{7\pi}{12}$
   $\frac{\pi}{4} + \frac{\pi}{3}$
   $\frac{3\pi}{12} + \frac{4\pi}{12}$
4. $-\frac{\pi}{12}$
   $\frac{\pi}{4} - \frac{\pi}{3}$

Now let's use the identity.

Examples: Find the exact values of the following trigonometric functions.

1. $\cos 75°$
   $\cos(30° + 45°)$
   $\cos 30° \cos 45° - \sin 30° \sin 45°$
   $\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$
2. $\cos \frac{\pi}{12}$
   $\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$
   $\cos\frac{\pi}{3} \cos\frac{\pi}{4} + \sin\frac{\pi}{3} \sin\frac{\pi}{4}$
   $\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$
   $\frac{\sqrt{2} + \sqrt{6}}{4}$

Now use the identity backwards to find the original angle.

Examples: Use the appropriate identities to simplify each expression.

1. $\cos 49° \cos 4° + \sin 49° \sin 4°$
   $\cos(49° - 4°)$
   $\cos 45°$
   $\frac{\sqrt{2}}{2}$
2. $\cos(x)\cos(-5x) + \sin(-x)\sin(5x)$
   $\cos x \cos 5x - \sin x \sin 5x$
   $\cos(x + 5x)$
   $\cos(6x)$

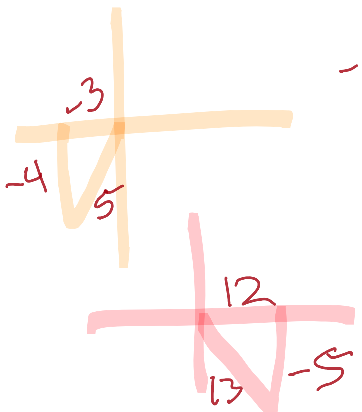
Now let's combine some identities to evaluate.

Examples: Find the exact values of the given trig function.

1. Find the exact value of $\cos(\alpha+\beta)$ if $\sin\alpha = -\frac{4}{5}$ and $\cos\beta = \frac{12}{13}$. α is in Quadrant III. β is in Quadrant IV.

$\cos\alpha\cos\beta - \sin\alpha\sin\beta$

$-\frac{3}{5}\cdot\frac{12}{13} - -\frac{4}{5}\cdot\frac{-5}{13}$



$\frac{-36-20}{65} = \frac{-56}{65}$

Cofunction Identities:

$\sin\left(\frac{\pi}{2}-u\right) = \cos u$ $\quad\cos\left(\frac{\pi}{2}-u\right) = \sin u$

$\tan\left(\frac{\pi}{2}-u\right) = \cot u$ $\quad\csc\left(\frac{\pi}{2}-u\right) = \sec u$

$\sec\left(\frac{\pi}{2}-u\right) = \csc u$ $\quad\cot\left(\frac{\pi}{2}-u\right) = \tan u$

OR

$\sin(90°-u) = \cos u$ $\quad\cos(90°-u) = \sin u$

$\tan(90°-u) = \cot u$ $\quad\csc(90°-u) = \sec u$

$\sec(90°-u) = \csc u$ $\quad\cot(90°-u) = \tan u$

Examples: Use the cofunction identities to find which trig functions are equivalent.

1. $\cos 30°$, so $\theta = 30°$

$\cos 30° = \sin(90°-\theta)$

$\cos 30° = \sin(90°-30°)$

$\cos 30° = \sin 60°$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

2. $\sec 20°$

$\csc(90°-20°)$

$\csc 70°$

3. $\cot\frac{4\pi}{3}$

$\tan\left(\frac{\pi}{2}-\frac{4\pi}{3}\right)$

$\tan\left(-\frac{5\pi}{6}\right)$

Examples: Simplify each identity.

1. $\sin(65°)\sin(5°) + \sin(-25°)\sin(-85°)$

$\cos(90°-65°)$, $\cos(90°-5°)$

$\cos(25°)\cos(85°) + \sin 25°\sin 85°$

$\cos(25°-85°)$

$\cos(-60°) = \boxed{\frac{1}{2}}$

2. $\cos\left(\frac{\pi}{2}-\theta\right)\cos(-\theta) - \sin(-\theta)\sin\left(\theta-\frac{\pi}{2}\right)$

$\cos\left(\frac{\pi}{2}-\theta\right)\cos(-\theta) - \sin\left(-\left(\frac{\pi}{2}-\theta\right)\right)$ $\quad\sin(-\theta)$

$\cos\left(\frac{\pi}{2}-\theta\right)\cos(\theta) - \sin\left(\frac{\pi}{2}-\theta\right)\sin\theta$

$\cos\left(\frac{\pi}{2}-\theta+\theta\right)$

$\cos\frac{\pi}{2} = \boxed{0}$