

Date: 4/10/24

Section: 8.4

Objective: I can use sum and difference of sine and tangent identities

What we did with cosine can also be done with sine and tangent.

## Sum and Difference of Sine

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$$

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$

## Sum and Difference of Tangent

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$$

Mnemonic Device:

Sine can't change signs

Mnemonic Device:

you tan and you tan then roll over 1 time and tan some more

Examples:

1. Find the exact value of
- $\sin 75^\circ$
- .

$$\sin(45^\circ + 30^\circ)$$

$$\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

Using the appropriate trig identity, simplify.

- 1.
- $\sin \frac{\pi}{12} \cos \frac{5\pi}{18} - \cos \frac{\pi}{12} \sin \frac{5\pi}{18}$

$$\sin\left(\frac{\pi}{12} - \frac{5\pi}{18}\right)$$

$$\frac{1 \cdot 3}{12 \cdot 18} - \frac{5 \cdot 2}{18 \cdot 12}$$

$$3 - 10$$

$$\sin\left(-\frac{7\pi}{36}\right)$$

or

$$-\sin\left(\frac{7\pi}{36}\right)$$

2. Find the exact value of
- $\tan \frac{5\pi}{12}$
- .

$$\tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$\tan \frac{\pi}{4} + \tan \frac{\pi}{6}$$

$$\frac{1 + \frac{1}{\sqrt{3}}}{1 - \tan \frac{\pi}{4} \tan \frac{\pi}{6}}$$

$$\frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$$

$$\frac{\frac{\sqrt{3} + 1}{\sqrt{3}}}{\frac{\sqrt{3} - 1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

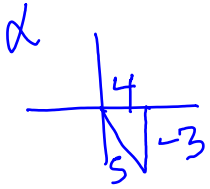
- 2.
- $\frac{\tan 58^\circ + \tan 31^\circ}{1 - \tan 58^\circ \tan 31^\circ}$

$$\tan(58^\circ + 31^\circ)$$

$$\tan(89^\circ)$$

Examples: Find the exact values of the given trig function.

1. Find the exact value of  $\sin(\alpha + \beta)$  if  $\sin \alpha = -\frac{3}{5}$  and  $\cos \beta = -\frac{1}{13}$ .  $\alpha$  is in Quadrant IV.  $\beta$  is in Quadrant III.



$$\sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$-\frac{3}{5} \cdot -\frac{1}{13} + \frac{4}{5} \cdot \frac{-2\sqrt{42}}{13}$$

$$\frac{3 - 8\sqrt{42}}{65}$$

