SM 3H

8.4 Sum & Difference Identities for Sine & Tangent

2023-2024

Name _____ Date _____ Period _____

Find the exact values of the following sums or differences.

1. $\dfrac{\pi}{4} + \dfrac{\pi}{12}$

2. $\dfrac{3\pi}{4} - \dfrac{\pi}{3}$

3. $\dfrac{\pi}{6} - \dfrac{\pi}{4}$

Express each given angle as $\alpha + \beta$ *or* $\alpha - \beta$ **, where** $\sin \alpha$ *and* $\sin \beta$ **are known exactly.**

4. $105°$

5. $-15°$

6. $-75°$

Use appropriate identities to find the exact value of each expression. No calculators!

7. $\sin\left(\dfrac{7\pi}{12}\right)$

8. $\sin\left(\dfrac{5\pi}{12}\right)$

9. $\tan\left(\dfrac{-13\pi}{12}\right)$

10. $\tan\left(\dfrac{7\pi}{12}\right)$

11. $\sin\left(-15°\right)$

12. $\tan\left(75°\right)$

Use the appropriate identities to simplify each expression to a single trigonometric function. If the trigonometric function is on the unit circle, write the ratio of the trigonometric function. Do not use a calculator. You can use the unit circle.

13. $\sin(23°)\cos(67°)+\cos(23°)\sin(67°)$

14. $\sin(2°)\cos(7°)-\cos(2°)\sin(7°)$

15. $\dfrac{\tan\left(\dfrac{\pi}{3}\right)-\tan\left(\dfrac{\pi}{5}\right)}{1+\tan\left(\dfrac{\pi}{3}\right)\tan\left(\dfrac{\pi}{5}\right)}$

16. $\dfrac{\tan\left(\dfrac{\pi}{9}\right)+\tan\left(\dfrac{\pi}{6}\right)}{1-\tan\left(\dfrac{\pi}{9}\right)\tan\left(\dfrac{\pi}{6}\right)}$

17. $\sin\left(-\dfrac{\pi}{2}\right)\cos\left(\dfrac{\pi}{5}\right)+\cos\left(\dfrac{\pi}{2}\right)\sin\left(-\dfrac{\pi}{5}\right)$

Solve each problem. Write answers as reduced fractions.

18. Find the exact value of $\sin(\alpha+\beta)$ if $\sin\alpha=\dfrac{3}{5}$ and $\sin\beta=\dfrac{5}{13}$, with α in quadrant II and β in quadrant I.

19. Find the exact value of $\sin(\alpha+\beta)$ if $\sin\alpha=\dfrac{7}{25}$ and $\sin\beta=\dfrac{-8}{17}$, with α in quadrant II and β in quadrant III.

20. Find the exact value of $\sin(\alpha + \beta)$ if $\sin \alpha = \frac{2}{3}$ and $\sin \beta = \frac{-1}{2}$, with α in quadrant I and β in quadrant III.

Write each expression as a function of α alone (only have an α *no* π).

21. $\sin(\alpha - \pi) \sin(\alpha - \pi)$

22. $\tan\left(\dfrac{\pi}{4} + \alpha\right)$

Verify that each equation is an identity.

23. $\dfrac{\sin(x + y)}{\sin x \cos y} = 1 + \cot x \tan y$

24. $\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta$

25. $\sin(2x) = 2\sin x \cos x$ **Hint: $2x = x + x$**

26. $\dfrac{\cos(\alpha - \beta)}{\sin(\alpha + \beta)} = \dfrac{1 + \tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$

27. Complete the sum and difference identities.

a) $\cos(x + y) = $ _____

b) $\cos(x - y) = $ _____

28. One of the acute angles of a right triangle is $26°$ and its hypotenuse is 38.6 inches. Find the lengths of its legs to the nearest tenth of an inch.

Verify that each equation is an identity.

29. $\cos\left(-\dfrac{\pi}{4} - 5t\right) = \cos\left(5t + \dfrac{\pi}{4}\right)$

30. $\sin(3\pi - 2\theta) = -\sin(2\theta - 3\pi)$