SM 3H

Date: 4/15/24 **Section:** 8.5

Objective: I can use double and half angle identities.

Double-Angle Identities:

$\sin(2x) = 2\sin x \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

$\cos(2x) = 1 - 2\sin^2 x$

$\cos(2x) = 2\cos^2 x - 1$

$\tan(2x) = \dfrac{2\tan x}{1 - \tan^2 x}$

$\sin(x+x) = \sin x \cos x + \cos x \sin x$

$\cos(x+x) = \cos x \cos x - \sin x \sin x$

$1 - \sin^2 x - \sin^2 x$

$\cos^2 x - 1 + \cos^2 x$

$\tan(x+x) = \dfrac{\tan x + \tan x}{1 - \tan x \tan x}$

Examples: Using the double-angle identities, find $\sin\left(\frac{2\pi}{3}\right)$, $\cos\left(\frac{2\pi}{3}\right)$, and $\tan\left(\frac{2\pi}{3}\right)$.

$\sin\left(2 \cdot \frac{\pi}{3}\right)$

$2\sin\frac{\pi}{3}\cos\frac{\pi}{3}$

$\cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{3}}{2}$

$\cos\left(2 \cdot \frac{\pi}{3}\right)$

$2\left(\cos\frac{\pi}{3}\right)^2 - 1$

$2\left(\frac{1}{2}\right)^2 - 1$

$2\left(\frac{1}{4}\right) - 1$

$\frac{1}{2} - 1$

$-\frac{1}{2}$

$\tan\left(2 \cdot \frac{\pi}{3}\right)$

$\dfrac{2\tan\frac{\pi}{3}}{1 - \left(\tan\frac{\pi}{3}\right)^2}$

$\dfrac{2 \cdot \sqrt{3}}{1 - \sqrt{3}^2}$

$\dfrac{2\sqrt{3}}{-2} = \boxed{-\sqrt{3}}$

Example: Verify using the double-angle identities.

1. $\cos(3x) = \cos^3 x - 3\cos x \sin^2 x$

$\cos(2x + x) =$

$\cos 2x \cos x - \sin 2x \sin x =$

$(\cos^2 x - \sin^2 x)\cos x - 2\sin x \cos x \sin x =$

$\cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x =$

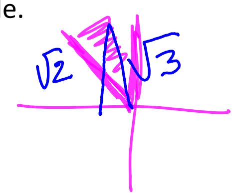
$\cos^3 x - 3\sin^2 x \cos x = \cos^3 x - 3\cos x \sin^2 x$

Example: Find $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$ given the following double-angle.

1. $\cos(2\alpha) = -\frac{1}{3}$, if $\pi < 2\alpha < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{2\alpha}{2} < \frac{3\pi}{2 \cdot 2}$

$\frac{\pi}{2} < \alpha < \frac{3\pi}{4}$



$1 - 2\sin^2\alpha = -\frac{1}{3}$

$-2\sin^2\alpha = -\frac{4}{3}$

$\sqrt{(\sin\alpha)^2} = \sqrt{\frac{2}{3}}$

$\sin\alpha = \sqrt{\frac{2}{3}}$

$\sqrt{\sqrt{3}^2 - \sqrt{2}^2} = 1$

$\cos\alpha = \frac{-1}{\sqrt{3}}$

$\tan\alpha = -\sqrt{2}$

Half-Angle Identities:

$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$

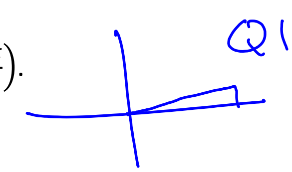
$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$

$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

$\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1+\cos x}$

$\tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x}$

Examples: Using the half-angle identities, find $\sin\left(\frac{\pi}{8}\right)$, $\cos\left(\frac{\pi}{8}\right)$, and $\tan\left(\frac{\pi}{8}\right)$.



$\sin\left(\frac{\frac{\pi}{4}}{2}\right)$

$\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}$

$\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$

$\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{2}}$

$\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

$\left(\cos\frac{\frac{\pi}{4}}{2}\right) = \frac{\sqrt{2+\sqrt{2}}}{2}$

$\tan\left(\frac{\frac{\pi}{4}}{2}\right) = \frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$

Example: Verify using the double-angle identities.

1. $\sin^2\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right) = \frac{\sin^2 x}{4}$

$$\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2 \cdot \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 =$$

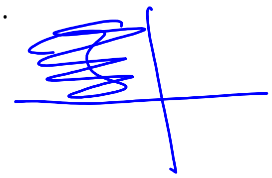
$$\frac{1-\cos x}{2} \cdot \frac{1+\cos x}{2} =$$

$$\frac{1-\cos^2 x}{4} =$$

$$\frac{\sin^2 x}{4} = \frac{\sin^2 x}{4}$$

Example: Find $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$ given the following half-angle.

1. $\sin\left(\frac{\alpha}{2}\right) = \frac{4}{5}$, if $\left(\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}\right)2$ $\quad \frac{\pi}{2} < \alpha < \pi$



$$\sqrt{\frac{1-\cos\alpha}{2}} = \frac{4}{5}$$

$$\frac{1-\cos\alpha}{2} = \frac{16}{25}$$

$$1-\cos\alpha = \frac{32}{25}$$

$$\cos\alpha = \frac{-7}{25}$$

$$\sin\alpha = \frac{24}{25} \qquad \tan\alpha = \frac{24}{-7}$$