SM 3H

Date: 4/17/24 **Section:** 8.6

Objective: I can use trig identities to solve trig equations.

Review:

1. Find all the angles that satisfy the following function.

$2\cos x + 1 = 0$ See Peter Pan

Steps for solving trig equations:

1. Get the trig function by itself. Sometimes this means you need to factor. Sometimes this means you need to use an identity in place of a trig function so that all of the trig functions are the same.

2. Using "U" substitution, find all the angles that work by drawing the picture if it is on the unit circle or using a calculator.

3. Write the answers as equations equal to what is inside the trig function in the original equation.

Example: original equation is $\cos 2x = \frac{1}{2}$ then the equations will be $2x = \frac{\pi}{6} + 2\pi k$ and $2x = \frac{11\pi}{6} + 2\pi k$

4. Solve the answer equations (equations from step 3).

5. Use the equations from step 4 to find the angles that work in the given interval.

6. Remember to check answers because taking a square root or squaring can give you extraneous answers.

Examples: Find all angles in the interval [0, 2π) that satisfy each equation. Round to the nearest hundredth.

1. $\sin(2x) = \cos x$

$2\sin x \cos x = \cos x$

$2\sin x \cos x - \cos x = 0$

$\cos x\,(2\sin x - 1) = 0$

$\cos x = 0 \qquad \sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

2. $\left(\sin\alpha - \cos\alpha = \frac{1}{\sqrt{2}}\right)^2$

$\sin^2\alpha - 2\sin\alpha\cos\alpha + \cos^2\alpha = \frac{1}{2}$

$1 - \sin(2\alpha) = \frac{1}{2}$

$+\sin(2\alpha) = +\frac{1}{2}$

$\frac{2\alpha}{2} = \frac{\frac{\pi}{6}}{2} + \frac{2\pi k}{2}$

$\frac{2\alpha = \frac{5\pi}{6} + 2\pi k}{2}$

$\sin u = \frac{1}{2}$

$u = \frac{\pi}{6}, \frac{5\pi}{6}$

$\alpha = \frac{\pi}{12} + \pi k$

$\alpha = \frac{5\pi}{12} + \pi k$

$\frac{\pi}{12} + \frac{12\pi}{12}$

$\frac{5\pi}{12} + \frac{12\pi}{12}$

$\alpha = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

Find all angles in the interval [0˚, 360˚) that satisfy each equation. Round approximations to the nearest tenth of a degree.

3. $\sin(2\theta) = \frac{\sqrt{3}}{2}$

$\frac{2\theta = 60^\circ + 360^\circ k}{2}$

$\theta = 30^\circ + 180^\circ k$

$\frac{2\theta = 120^\circ + 360^\circ k}{2}$

$\theta = 60^\circ + 180^\circ k$

$\sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

$60^\circ \quad 120^\circ$

$\theta = 30^\circ, 60^\circ, 210^\circ, 240^\circ$

4. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

$2\left(\frac{x}{2} = 30^\circ + 360^\circ k\right)$

$x = 60^\circ + 720^\circ k$

$2\left(\frac{x}{2} = 330^\circ + 360^\circ k\right)$

$x = 660^\circ + 720^\circ k$

$\cos u = \frac{\sqrt{3}}{2}$

$u = 30^\circ, 330^\circ$

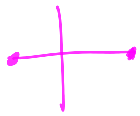
$x = 60^\circ$

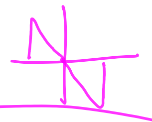
5. $\sin x \tan x + \sin x = 0$

$\sin x(\tan x + 1) = 0$

$\sin(x) = 0 \qquad \tan x = -1$

$x = 0° + 360°k$





$x = 0°, 180°, 135°, 315°$

6. $\cos(2x)\cos x - \sin(2x)\sin x = \frac{\sqrt{3}}{2}$

$\cos(2x + x) = \frac{\sqrt{3}}{2}$

$\cos 3x = \frac{\sqrt{3}}{2}$

$\frac{3x = 30° + 360°k}{3} \qquad \frac{3x = 330° + 360°k}{3}$

$x = 10° + 120°k \qquad x = 110° + 120°k$

$x = 10°, 130°, 250°, 110°, 230°, 350°$