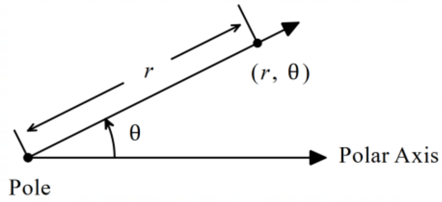
SM 3H

Date: **Section:**

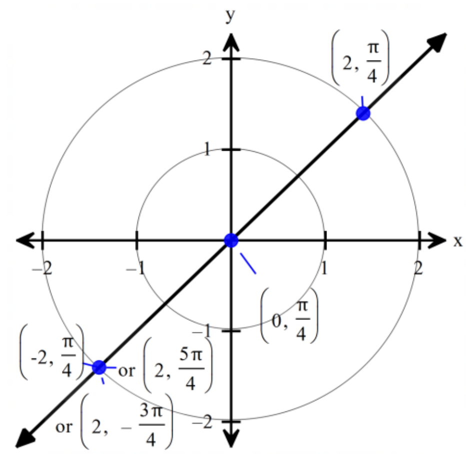
Objective:



The polar coordinate system is based on a fixed point called the **pole** and a fixed axis called the **polar axis.** Points are represented by ordered pairs in the form (r,θ), where r is the **directed distance** from the pole and θ is an angle whose initial side is the polar axis and whose terminal side contains the point.

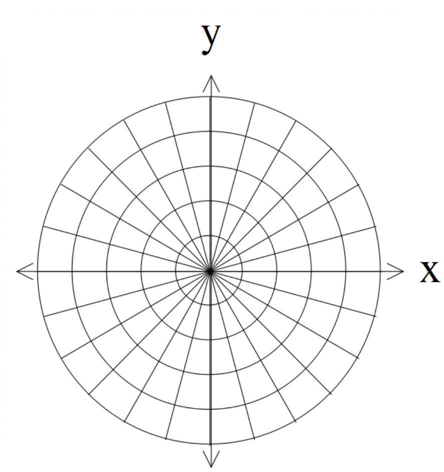
Typically, we choose the origin as the pole and the positive x-axis as the polar axis.

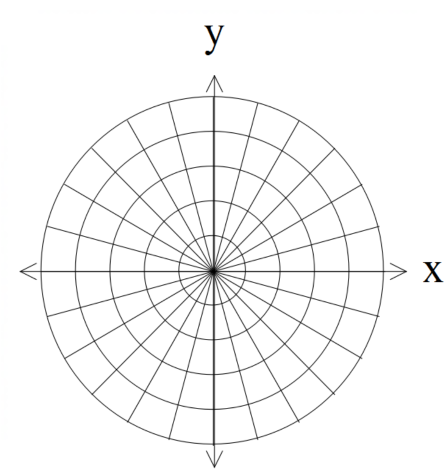
*To graph $(-r,\theta)$, you move in the opposite direction you would move to graph (r,θ).



Polar coordinates are not unique. The points $\left(-2,\frac{\pi}{4}\right)$, $\left(2,\frac{5\pi}{4}\right)$, and $\left(2,-\frac{3\pi}{4}\right)$ all name the same point.

Examples: Plot the points whose polar coordinates are given.
$A\left(3,\frac{\pi}{3}\right)$, $B\left(-1,\frac{\pi}{6}\right)$, $C\left(2,-\frac{7\pi}{4}\right)$, $D\left(-5,-\frac{3\pi}{4}\right)$, $E\left(4,\frac{\pi}{2}\right)$, $F\left(-3,\frac{2\pi}{3}\right)$





Every Polar coordinate has an equivalent rectangular coordinate.

Polar-Rectangular Conversion Rules

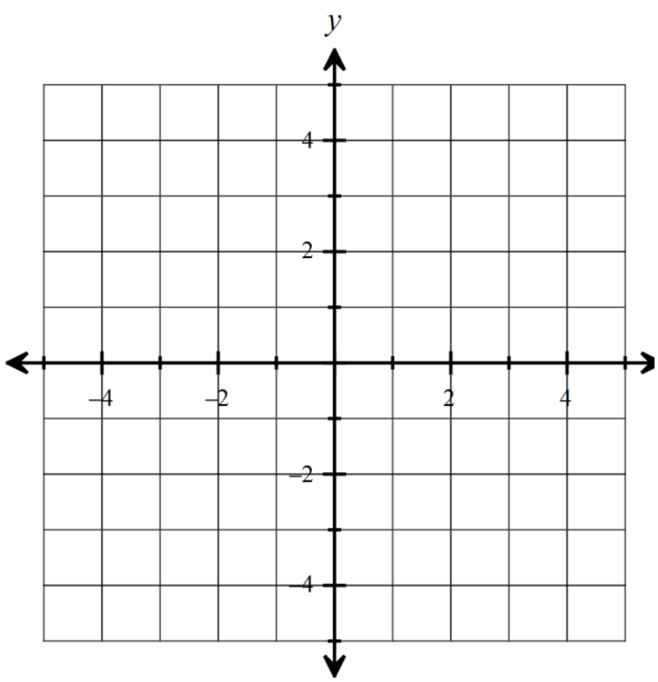
- To convert (r,θ) to rectangular coordinates (x,y), use $x=$ ________ and $y=$ ________.
- To convert (x,y) to polar coordinates (r,θ), use $r=$ ________ and any angle θ in standard position whose terminal side contains (x,y).

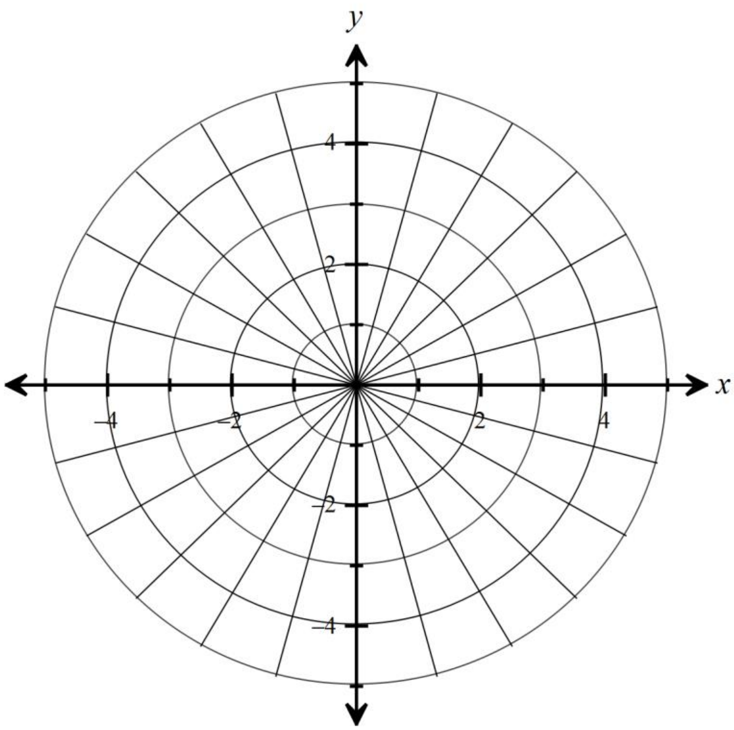
Example:

Change $\left(5, \frac{\pi}{3}\right)$ from polar to rectangular.

Then graph the rectangular coordinates on the coordinate plane.

Change $\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$ from rectangular to polar. Then graph the polar coordinates on the polar graph.





Notice they are in the same spot of both graphs. If you laid one on top of the other, the points would be on top of each other.

Examples:

a) Convert $(3, 45°)$ to rectangular coordinates.

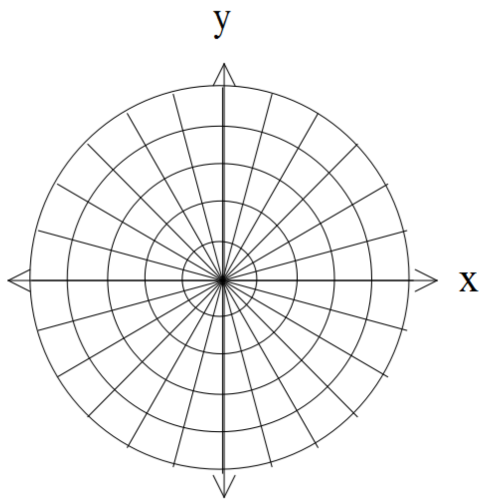
b) Convert $\left(-2, 2\sqrt{3}\right)$ to polar coordinates.

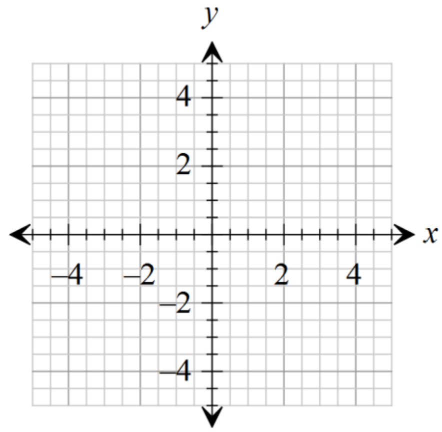
Graphing Polar Equations

Graphing Calculator link https://www.desmos.com/calculator

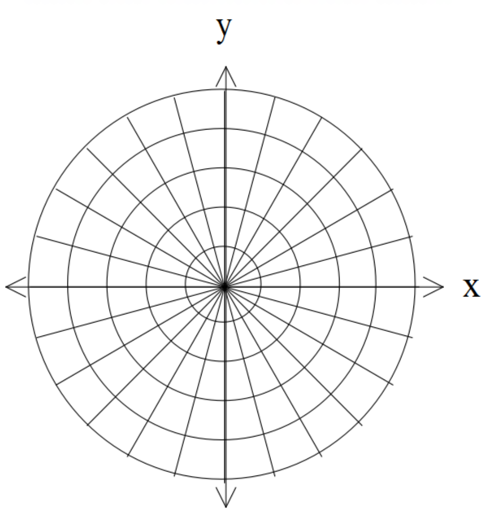
Examples: Sketch the graphs of the following:

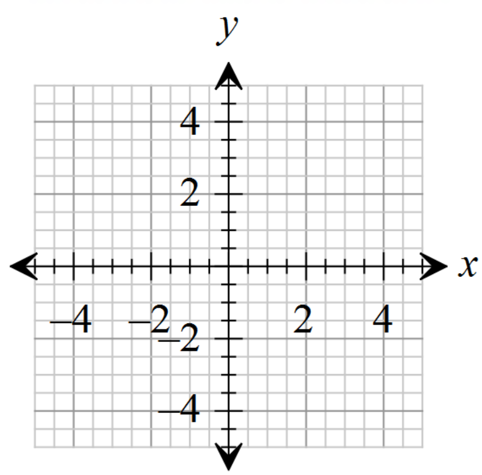
a) $r = 4\sin\theta$



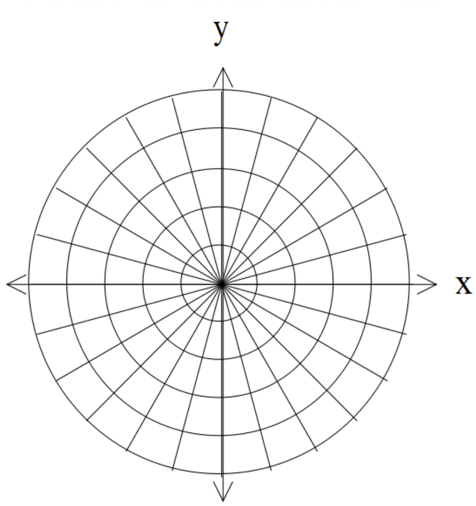


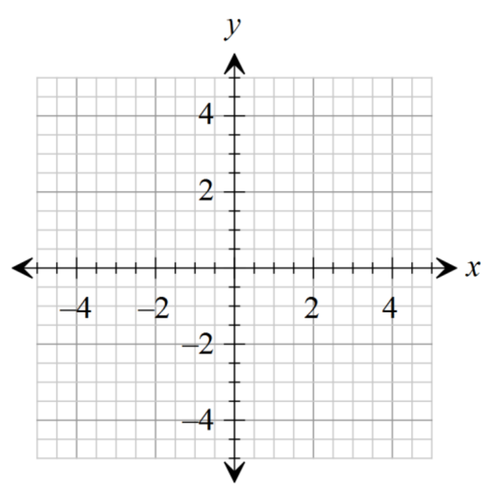
b) $r = \cos(2\theta)$





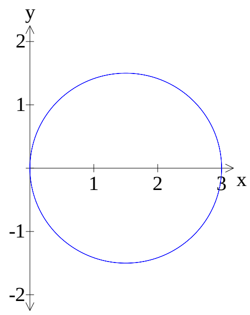
c) $r = 3 + 4\sin\theta$



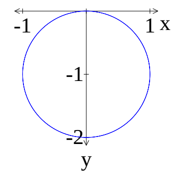


1) Lines through the origin are of the form $\theta = \alpha$.
Vertical lines are of the form $r = a\sec\theta$.
Horizontal lines are of the form $r = a\csc\theta$.

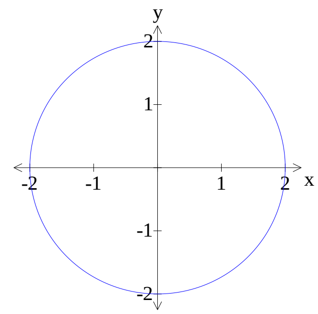
2) Circles come in three forms: $r = a\cos\theta$, $r = a\sin\theta$, and $r = a$.



$r = 3\cos\theta$

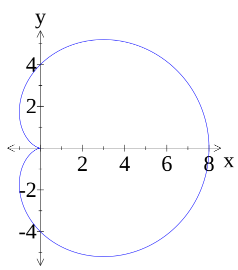


$r = -2\sin\theta$

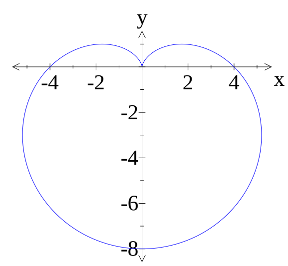


$r = 2$

3) Cardioids have the form $r = a \pm a\cos\theta$ or $r = a \pm a\sin\theta$.
Cardioids pass through the pole.

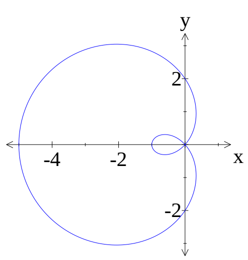


$r = 4 + 4\cos\theta$

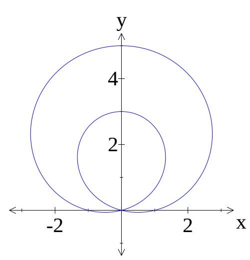


$r = 4 - 4\sin\theta$

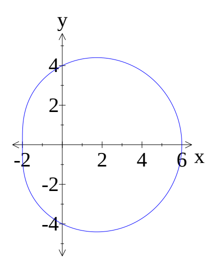
4) Limaçons have the form $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$.
Limaçons have an inner loop if $0 < a < b$ and have no inner loop if $0 < b < a$.
The graph of a limaçon with an inner loop passes through the pole twice.
The graph of a limaçon with no inner loop does not pass through the pole.



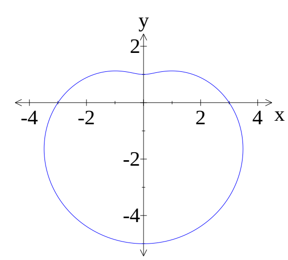
$r = 2 - 3\cos\theta$



$r = 1 + 4\sin\theta$

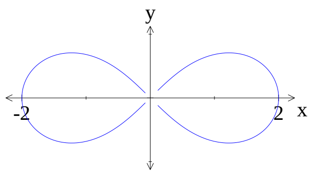


$r = 4 + 2\cos\theta$

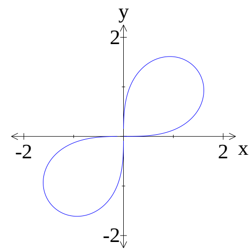


$r = 3 - 2\sin\theta$

5) Lemniscates have the form $r^2 = a^2 \cos(2\theta)$ or $r^2 = a^2 \sin(2\theta)$.

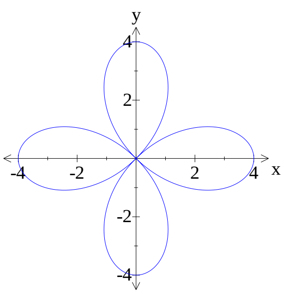


$r^2 = 4\cos(2\theta)$

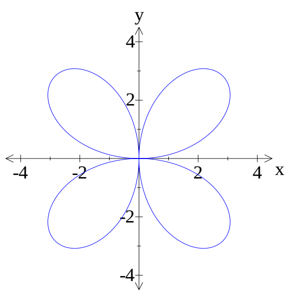


$r^2 = 4\sin(2\theta)$

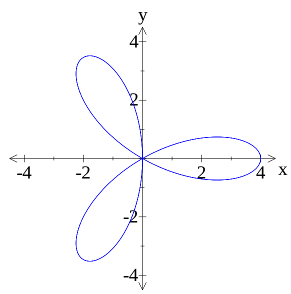
6) Roses have the form $r = a\cos(n\theta) + b$ and $r = a\sin(n\theta) + b$.
If n is even, there are $2n$ loops in the rose.
If n is odd, there are n loops in the rose.



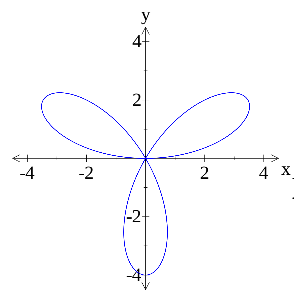
$r = 4\cos(2\theta)$



$r = 4\sin(2\theta)$

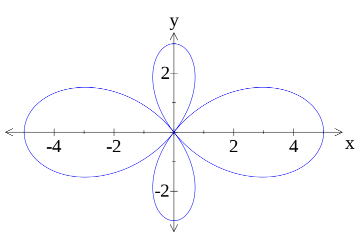


$r = 4\cos(3\theta)$

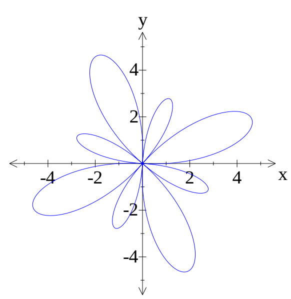


$r = 4\sin(3\theta)$

Examples:



$r = 4\cos(2\theta) + 1$



$r = 4\sin(4\theta) + 1$

Play around with your calculator and see the cool things you can create by changing the numbers.

Convert equations from polar to rectangular form.

a) Convert $r = 3\sin\theta$ to a rectangular equation.

b) Convert $r = \dfrac{4}{1+\sin\theta}$ to a rectangular equation.

c) Convert $r = 5\sec\theta$ to a rectangular equation.

d) Convert $r = 5$ to a rectangular equation.

Examples: Convert equations from rectangular to polar form.

a) Convert $y = 7$ to a polar equation.

b) Convert $y = -2x + 5$ to a polar equation.

c) Convert $x^2 + (y-1)^2 = 1$ to a polar equation.