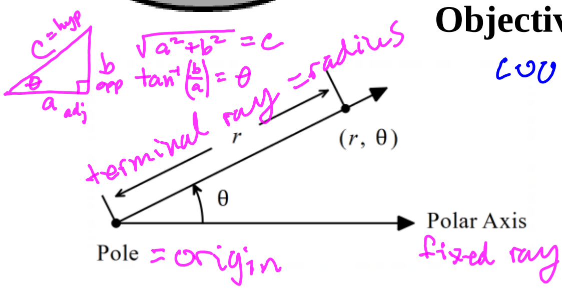
SM 3H

Date: 4/25/24 **Section:** 9.1

Objective: I can convert rectangular & polar coordinates ~~and~~ equations.

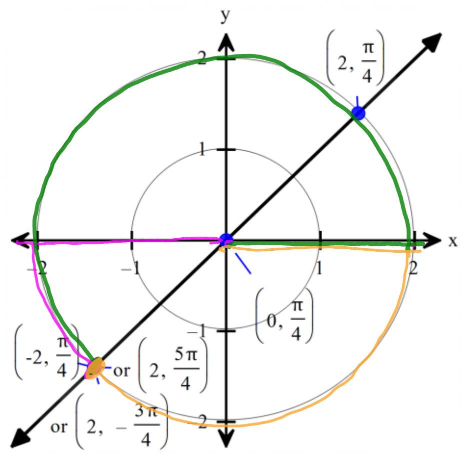


The polar coordinate system is based on a fixed point called the **pole** and a fixed axis called the **polar axis.** Points are represented by ordered pairs in the form (r,θ), where r is the **directed distance** from the pole and θ is an angle whose initial side is the polar axis and whose terminal side contains the point.

Typically, we choose the origin as the pole and the positive x-axis as the polar axis.

radius, angle

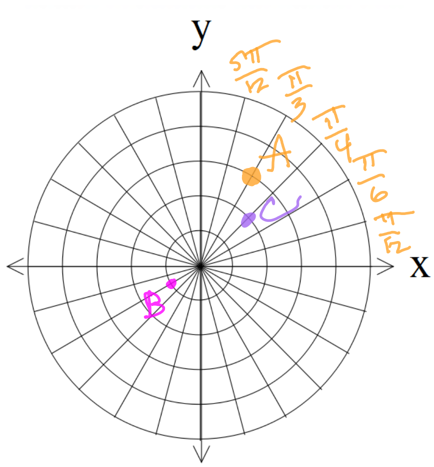
*To graph $(-r,\theta)$, you move in the opposite direction you would move to graph (r,θ).

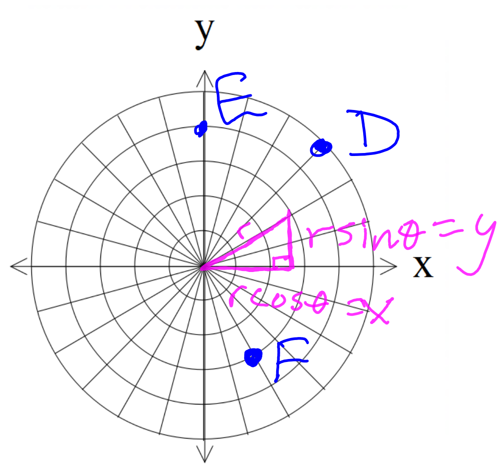


Polar coordinates are not unique. The points $\left(-2,\frac{\pi}{4}\right)$, $\left(2,\frac{5\pi}{4}\right)$, and $\left(2,-\frac{3\pi}{4}\right)$ all name the same point.

Examples: Plot the points whose polar coordinates are given.

$A\left(3,\frac{\pi}{3}\right)$, $B\left(-1,\frac{\pi}{6}\right)$, $C\left(2,-\frac{7\pi}{4}\right)$, $D\left(-5,-\frac{3\pi}{4}\right)$, $E\left(4,\frac{\pi}{2}\right)$, $F\left(-3,\frac{2\pi}{3}\right)$





Every Polar coordinate has an equivalent rectangular coordinate.

Polar-Rectangular Conversion Rules

- To convert (r,θ) to rectangular coordinates (x,y), use $x =$ $r\cos\theta$ and $y =$ $r\sin\theta$.
- To convert (x,y) to polar coordinates (r,θ), use $r =$ $\sqrt{x^2+y^2}$ and any angle θ in standard position whose terminal side contains (x,y). $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Example:

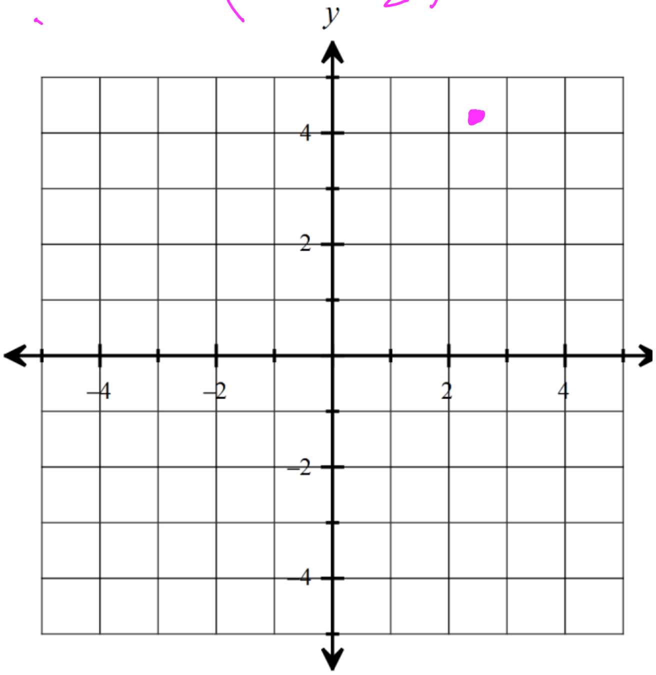
Change $\left(5, \frac{\pi}{3}\right)$ from polar to rectangular.

Then graph the rectangular coordinates on the coordinate plane.

$x = 5\cos\frac{\pi}{3} = 5\left(\frac{1}{2}\right) = \frac{5}{2}$

$y = 5\sin\frac{\pi}{3} = 5\left(\frac{\sqrt{3}}{2}\right) = \frac{5\sqrt{3}}{2}$

$\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

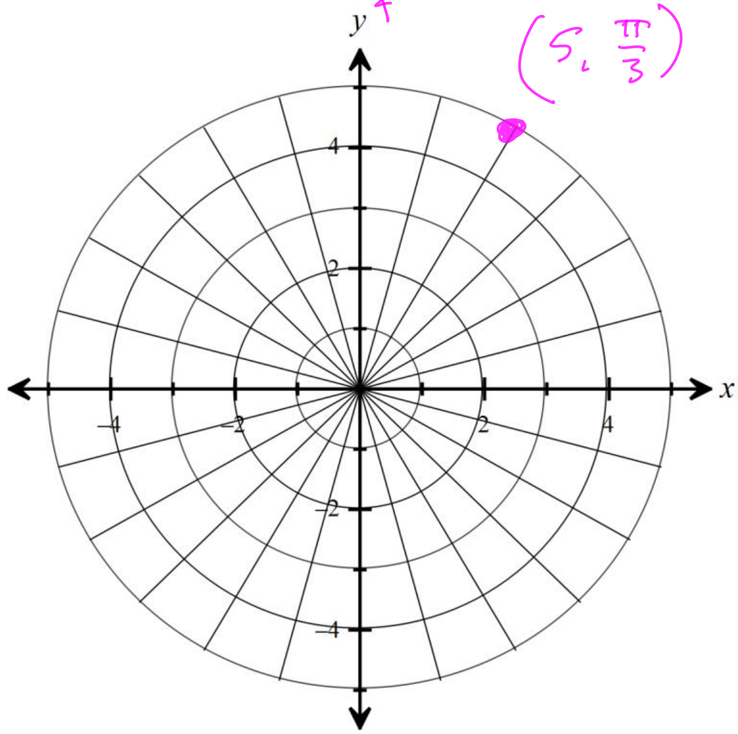


Change $\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$ from rectangular to polar. Then graph the polar coordinates on the polar graph.

$r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2} = \sqrt{\frac{25}{4} + \frac{75}{4}} = 5$

$\theta = \tan^{-1}\left(\frac{\frac{5\sqrt{3}}{2}}{\frac{5}{2}}\right) = \frac{\pi}{3}$

$\left(5, \frac{\pi}{3}\right)$



Notice they are in the same spot of both graphs. If you laid one on top of the other, the points would be on top of each other.

Examples:

a) Convert $(3, 45°)$ to rectangular coordinates.

$x = 3\cos 45° = 3\left(\frac{\sqrt{2}}{2}\right) = \frac{3\sqrt{2}}{2}$

$y = 3\sin 45° = \frac{3\sqrt{2}}{2}$

$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

b) Convert $(-2, 2\sqrt{3})$ to polar coordinates.

QII

$r = \sqrt{2^2 + (2\sqrt{3})^2} = 4$

$\theta = \tan^{-1}\left(\frac{\cancel{2}\sqrt{3}}{-\cancel{2}}\right) = \frac{2\pi}{3}$ $\quad \frac{\sqrt{3}}{-1}$

$\left(4, \frac{2\pi}{3}\right)$

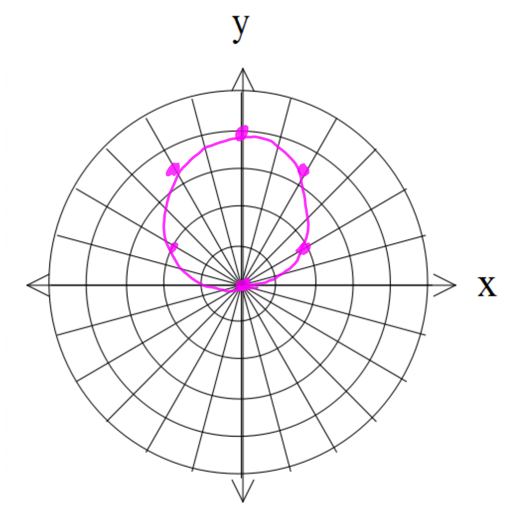
Graphing Polar Equations

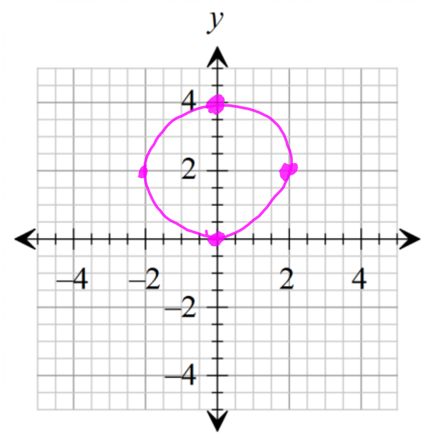
Graphing Calculator link https://www.desmos.com/calculator

Examples: Sketch the graphs of the following:

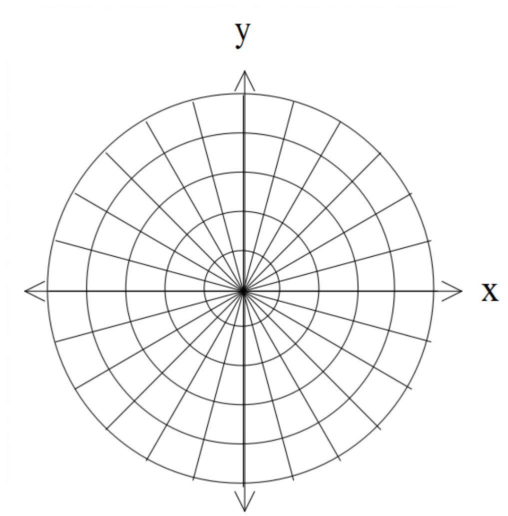
a) $r = 4\sin\theta$

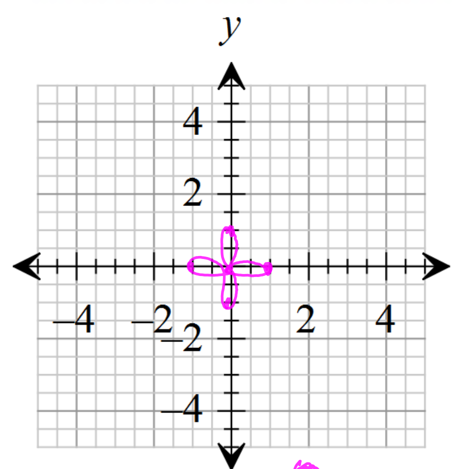
r	θ
0	0
2	$\frac{\pi}{6}$
$3.5 \approx 2\sqrt{3}$	$\frac{\pi}{3}$
4	$\frac{\pi}{2}$
$2\sqrt{3}$	$\frac{2\pi}{3}$
2	$\frac{5\pi}{6}$
0	π



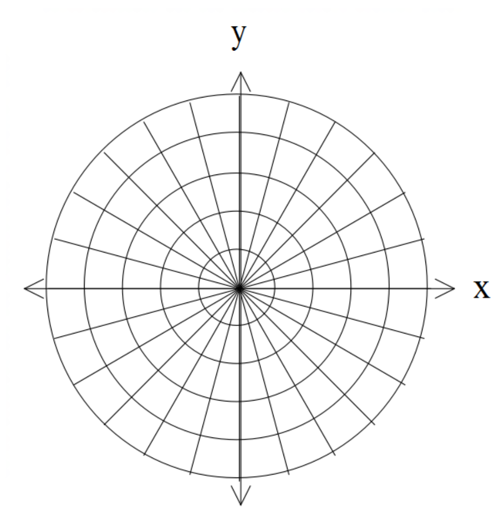


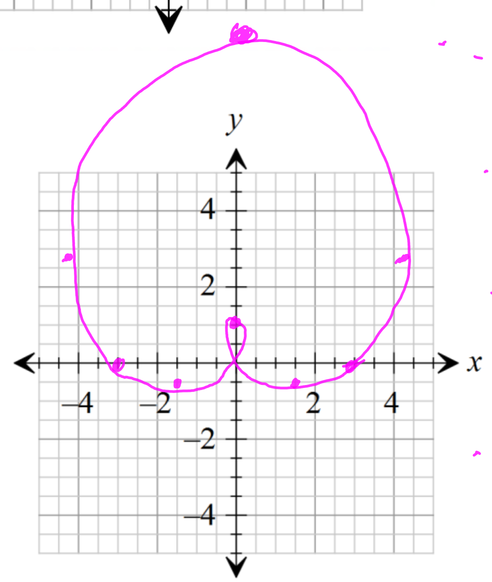
b) $r = \cos(2\theta)$





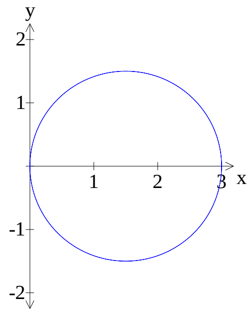
c) $r = 3 + 4\sin\theta$



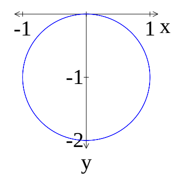


1) Lines through the origin are of the form $\theta = \alpha$.
Vertical lines are of the form $r = a\sec\theta$.
Horizontal lines are of the form $r = a\csc\theta$.

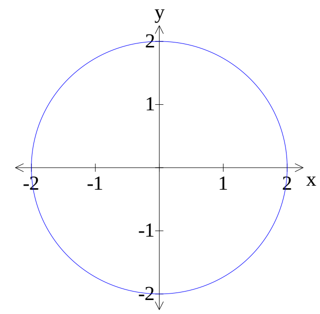
2) Circles come in three forms: $r = a\cos\theta$, $r = a\sin\theta$, and $r = a$.



$r = 3\cos\theta$

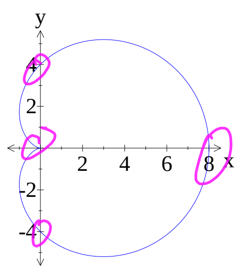


$r = -2\sin\theta$

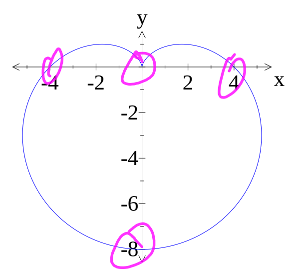


$r = 2$

3) Cardioids have the form $r = a \pm a\cos\theta$ or $r = a \pm a\sin\theta$.
Cardioids pass through the pole.

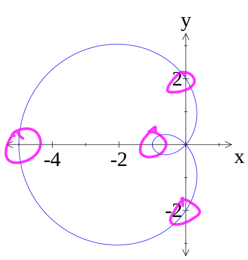


$r = 4 + 4\cos\theta$

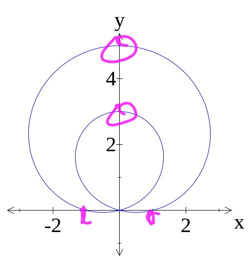


$r = 4 - 4\sin\theta$

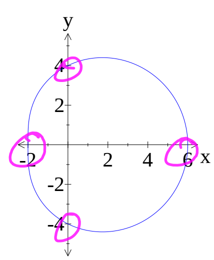
4) Limaçons have the form $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$.
Limaçons have an inner loop if $0 < a < b$ and have no inner loop if $0 < b < a$.
The graph of a limaçon with an inner loop passes through the pole twice.
The graph of a limaçon with no inner loop does not pass through the pole.



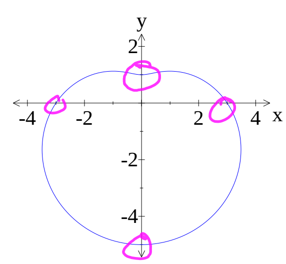
$r = 2 - 3\cos\theta$



$r = 1 + 4\sin\theta$

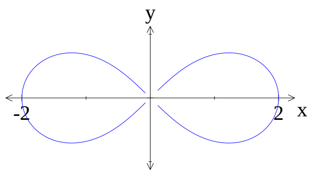


$r = 4 + 2\cos\theta$

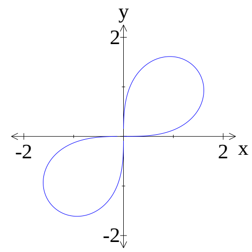


$r = 3 - 2\sin\theta$

5) Lemniscates have the form $r^2 = a^2\cos(2\theta)$ or $r^2 = a^2\sin(2\theta)$.



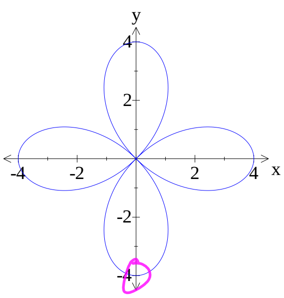
$r^2 = 4\cos(2\theta)$



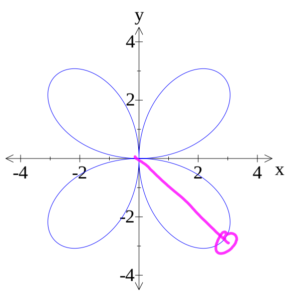
$r^2 = 4\sin(2\theta)$

6) Roses have the form $r = a\cos(n\theta) + b$ and $r = a\sin(n\theta) + b$.
If n is even, there are $2n$ loops in the rose.
If n is odd, there are n loops in the rose.

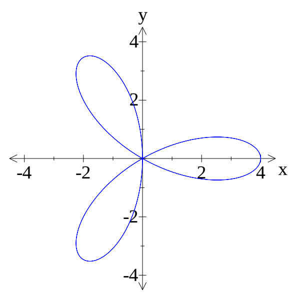
Examples:



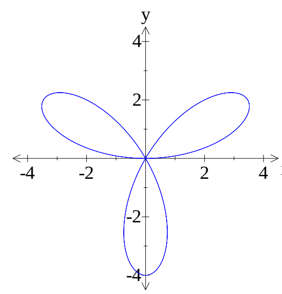
$r = 4\cos(2\theta)$



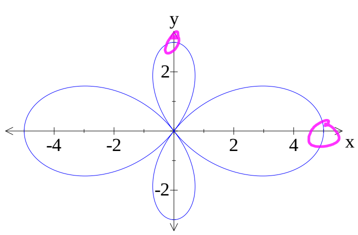
$r = 4\sin(2\theta)$



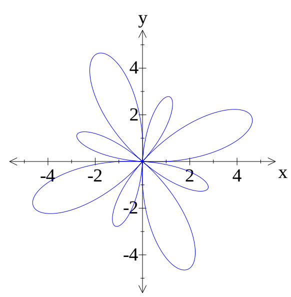
$r = 4\cos(3\theta)$



$r = 4\sin(3\theta)$



$r = 4\cos(2\theta) + 1$



$r = 4\sin(4\theta) + 1$

Play around with your calculator and see the cool things you can create by changing the numbers.

$r^2 = x^2 + y^2 \qquad x = r\cos\theta \qquad y = r\sin\theta$

Convert equations from polar to rectangular form.

a) Convert $(r = 3\sin\theta)r$ to a rectangular equation.

$r^2 = 3r\sin\theta$

$x^2 + y^2 = 3y$

b) Convert $r = \dfrac{4}{1+\sin\theta}$ to a rectangular equation.

$r + r\sin\theta = 4$

$r + y = 4$

$r^2 = (4-y)^2$

$r^2 = 16 - 8y + y^2$

$x^2 + \cancel{y^2} = 16 - 8y + \cancel{y^2}$

$x^2 = 16 - 8y$

c) Convert $r = 5\sec\theta$ to a rectangular equation.

$r = \dfrac{5}{\cos\theta}$

$r\cos\theta = 5$

$x = 5$

d) Convert $r^2 = 5^2$ to a rectangular equation.

$r^2 = 25$

$x^2 + y^2 = 25$

Examples: Convert equations from rectangular to polar form.

a) Convert $y = 7$ to a polar equation.

$r\sin\theta = 7$

$r = \dfrac{7}{\sin\theta} = 7\csc\theta$

b) Convert $y = -2x + 5$ to a polar equation.

$r\sin\theta = -2r\cos\theta + 5$

$r\sin\theta + 2r\cos\theta = 5$

$r(\sin\theta + 2\cos\theta) = 5$

$r = \dfrac{5}{\sin\theta + 2\cos\theta}$

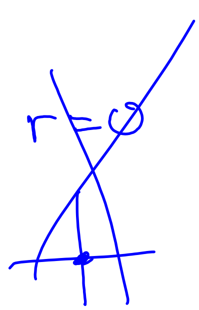
c) Convert $x^2 + (y-1)^2 = 1$ to a polar equation.

$x^2 + y^2 - 2y + 1 = 1$

$r^2 - 2r\sin\theta = 0$

$r(r - 2\sin\theta) = 0$

$r = 0$



$r = 2\sin\theta$