SM 3H

**Date:** **Section:**

**Objective:**

Sometimes, it is convenient to express both *x* and *y* as functions of a third variable, *t*. If $f(t)$ and $g(t)$ are both functions of *t*, where *t* is some interval of real numbers, then the equations $x = f(t)$ and $y = g(t)$ are called ______________ ______________**.** The variable $t$ is called the __________________**.** If we think of *t* as time, then we know when each point of the graph is plotted.

Algebra is really about relationships. How are things connected? Do they move together, or apart, or maybe they're completely independent?

Normal equations assume an "input to output" connection. That is, we take an input ($x = 3$), plug it into the relationship ($y = x^2$), and observe the result ($y = 9$).

But is that the only way to see a scenario? The setup $y = x^2$ implies that *y* only moves because of *x*. But it could be that y just coincidentally equals $x^2$, and some hidden factor is changing them both (the factor changes *x* to 3 while also changing *y* to 9).

As a real world example: For every degree above 70, our convenience store sells *x* bottles of sunscreen and $x^2$ pints of ice cream.

We *could* write the algebra relationship like this:

ice cream = (suncscreen)$^2$

And it's correct… but misleading!

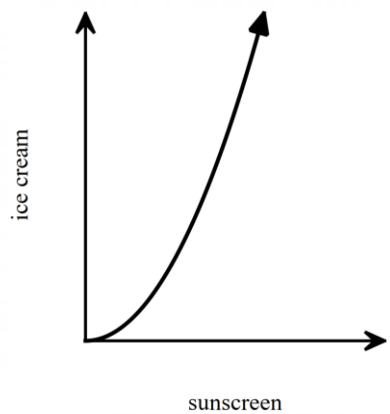
The equation implies sunscreen directly changes the demand for ice cream, when it's the hidden variable (temperature) that changed them both!

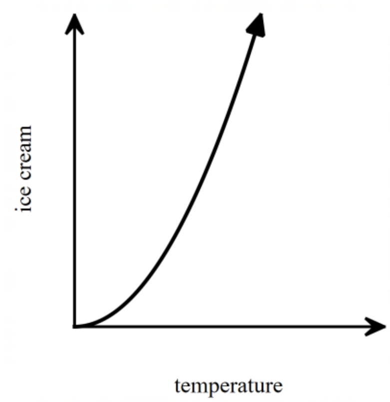
It's much better to write two separate equations

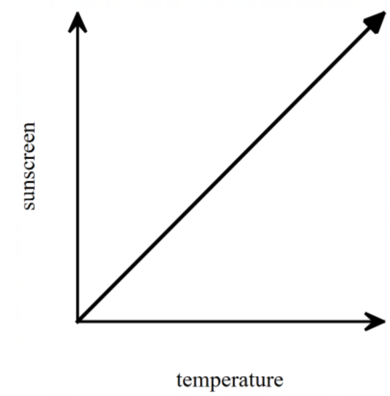
Sunscreen = temperature – 70

Ice cream = (temperature – 70)$^2$

that directly point out the causality. The ideas "temperature impacts ice cream" and "temperature impacts sunscreen" clarify the situation, and we lose information by trying to factor away the common "temperature" portion. Parametric equations get us closer to the real-world relationship.



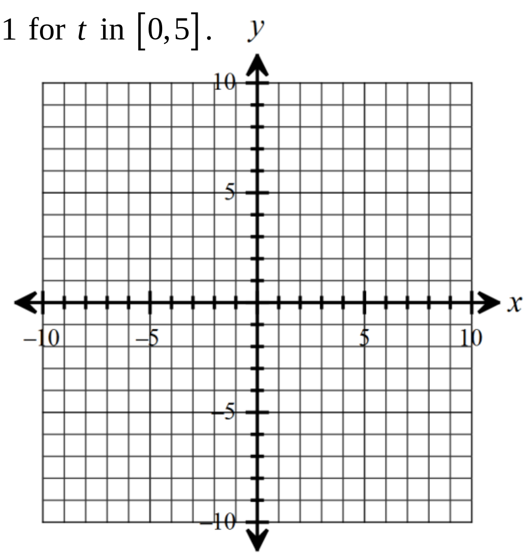

**Since temperature affects both ice cream sales and sunscreen sales, temperature is the parameter, that creates a relationship between ice cream sales and sunscreen sales.**

**Graphing Parametric Equations**

1. Make a *t*, *x*, *y* table for the two equations.
2. Plot the ordered pairs of values of *x* and *y*.
3. Mark the ________________ of the curve by using arrows to show the direction of the graph.

**Example:** Graph the parametric equations $x = t + 5$ and $y = 2t - 1$ for $t$ in $[0,5]$.

| *t* | *x* | *y* |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |



## Using a graphing calculator, graph each pair of parametric equations in the rectangular coordinate system.

**There are several website to help you graph with your calculator. Here is a link to a video using a TI 83 or TI 84. Copy and paste in the search bar on the internet and watch the video.**

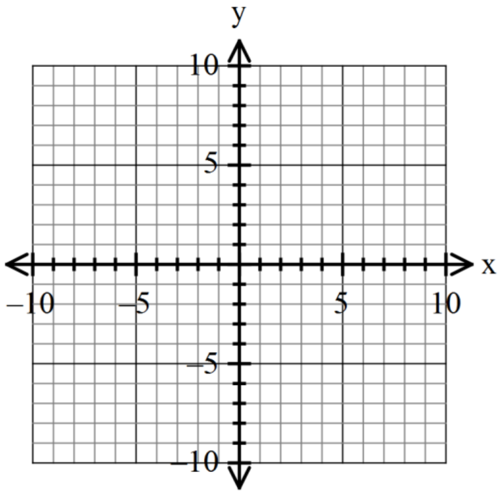
https://www.google.com/search?q=how+to+graph+parametric+equations+on+ti-84+plus+ce&rlz=1C1CHFX_enUS526US526&oq=how+to+graph+parametric+equations&aqs=chrome.2.69i57j0l7.14735j0j7&sourceid=chrome&ie=UTF-8#kpvalbx=_a9uYXuvVGJTNtQaxtabwAg35

Another great idea is to go to GeoGebra and use that calculator.(Very easy to use.)

https://www.geogebra.org/m/yJNhQMQa

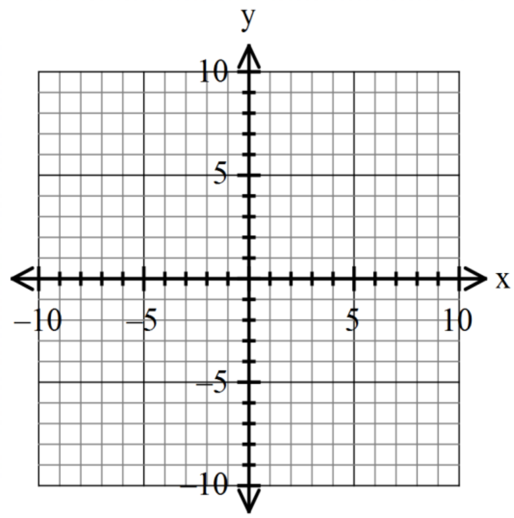
Example 1:

$x = t + 3,\ y = -4t - 1,\ \text{for } -4 \le t \le 2$




Example 2:

$x = t - 2, \quad y = t^2 + 2, \quad for\ t\ in\ (-\infty, \infty)$




### Eliminating the Parameter

1. Set one equation equal to $t$.
2. Substitute that equation in for $t$ in the other equation.
3. Sometimes it is more convenient to use a trigonometric identity to eliminate the parameter.

**Examples:** Eliminate the parameter and identify the graph of the parametric equation.

a) $x = 4t - 9,\ y = -t + 1,\ -\infty < t < \infty$

b) $x = 2\sqrt{t},\ y = 8t + 6,\ \ 0 \le t < \infty$

c) $x = 5\sin t,\ y = 5\cos t,\ -\infty < t < \infty$

d) $x = 2\sin\theta,\ y = 3\cos\theta,\ \ -\infty < \theta < \infty$