SM 3H

Date: 5/3/24 **Section:** 9.3

Objective: I can write parametric equations and solve parametric stories.

Types of parent equations:

linear: $y = mx + b$ quadratic: $y = ax^x + bx + c$

circle: $(x-h)^2 + (y-k)^2 = r^2$ parametric: $x = f(t), \quad y = g(t)$

Writing Parametric Equations for Line Segments

1. Write both parametric equations as linear functions: $x = m_1 t + b_1$, and $y = m_2 t + b_2$.
2. Substitute *x* and *t* values into the *x* equation to create a system of equations you can solve for m_1 and b_1.
3. Substitute *y* and *t* values into the *y* equation to create a system of equations you can solve for m_2 and b_2.

Examples:

Write parametric equations for the line segment starting at $(1,2)$ with $t = 0$ and ending at $(8,10)$ with $t = 1$.

x y

$1 = m_1(0) + b_1$

$1 = b_1$

$8 = m_1(1) + 1$

$m_1 = 7$

y

$2 = m_2(0) + b_2$

$2 = b_2$

$10 = m_2(1) + 2$

$m_2 = 8$

x y

$x = 7t + 1$

$y = 8t + 2$

$0 \le t \le 1$

Write parametric equations for the line segment starting at $(-2,4)$ with $t = 3$ and ending at $(5,-9)$ with $t = 7$. This type will be done in calculus, where the parameter doesn't start at 0.

Write parametric equations for the portion of the circle $x^2 + y^2 = 16$ that lies in the second quadrant.

Circle $r = 4$ center $(0,0)$

$x = r\cos\theta$

$y = r\sin\theta$

$x = 4\cos\theta$

$y = 4\sin\theta$

$\frac{\pi}{2} \le \theta \le \pi$

Write parametric equations for the circle with center (-4, 1) and radius 5 and $0 \le t \le 2\pi$.

$(x+4)^2 + (y-1)^2 = 25$

$x = 5\cos t - 4$

$y = 5\sin t + 1$

Writing Parametric Equations for a Polar Equation

Use the equations $x = r\cos\theta$ and $y = r\sin\theta$. Replace r to obtain the parametric equations. When converting polar equations to parametric equations, θ acts as the parameter.

Example: Write parametric equations for the polar equation $r = 3\cos\theta$.

diameter

Polar

$x = 3\cos\theta \cos\theta = 3\cos^2\theta$

$y = 3\cos\theta \sin\theta$

$0 \le \theta \le 2\pi$

rectangular

$x = 1.5\cos\theta + 1.5$

$y = 1.5\sin\theta$

$0 \le \theta \le 2\pi$

center = (1.5, 0)

radius = 1.5

The following problem involves the parametric equations for the path of a projectile.

$$x = v_0(\cos\theta)t \quad and \quad y = -16t^2 + v_0(\sin\theta)t + h_0$$

horizontal distance; = height

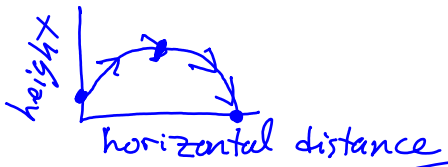
where θ is the angle of inclination of the projectile at the launch, v_0 is the initial velocity of the projectile in feet per second, and h_0 is the initial height of the projectile in feet.

Ms. Gordon went to a driving range where there are 3 levels with 5 spots across on each level. She hit a golf ball with an initial speed of 80 feet per second at an angle of 60° to the horizontal. She was on the second level which is 10 feet above the ground.

a) Find the parametric equations that describe the position of the ball as a function of time.

$x = (80\cos 60°)t = 80\left(\frac{1}{2}\right)t = 40t$

$y = -16t^2 + (80\sin 60°)t + 10 = -16t^2 + 40\sqrt{3}t + 10$



b) How long is the ball in the air?

$0 = -16t^2 + 40\sqrt{3}t + 10$ $\quad t \approx 4.47$ sec.

vertex

$x = \frac{-b}{2a}$

c) When is the ball at its maximum height? Determine the maximum height of the ball.

$x = \frac{-40\sqrt{3}}{2(-16)} \approx 2.17$ sec $\quad y = -16(2.17)^2 + 40\sqrt{3}(2.17) + 10$

$y = 85$ ft

d) Determine the distance the ball traveled.

$x = 40(4.47) \approx 178.8$ ft.